Unit 2 Packet

9/8 or 9/9

P. 1

Equations and Their Solutions

A LOT of time is spent in Algebra learning how to solve equations and then solving them for various purposes. So, it goes without saying that we really need to understand what it means for something to "solve" an equation. First, let's make sure we understand what an equation is.

Equation Definition

An equation is simply a statement about the equality of two expressions. In other words, anything that takes this form.

Expression #1 = Expression #2

Exercise #1: Which of the following is **not** an equation?

(1) $3+1=4+0$ $4=4$

(2) $x^2-2x=8$

(3) $2(4x+1)$

(4) $1+3=6$ $4\neq 6$

Equations can be either true, like (1) above, or false, like (4) above, depending on whether the two expression are equal (true) or not equal (false).

Exercise #2: Consider the equation $2x-8=10-x$

(a) Why can't you determine whether this equation is true or false?

It has a variable

(b) If $x=5$, will the equation be true? How can you tell?

$2(5)-8=10-5$
$10-8=5$
$2\neq 5$ no

(c) Show that $x=6$ makes the equation true. Remember to think very carefully always about your order of operations.

$2(6)-8=10-6$
$12-8=10-6$
$4=4$ ✓

Solutions to Equations

A value for a variable is called a **solution to the equation** if, when substituted into both expressions, results in the equation being true.

P. 2

This concept of the solution to an equation is **amazingly important**. It implies that you can always know when you have solved an equation correctly. As long as you can check the truth of the equation with arithmetic, then you will know if your value (or x often) is correct.

Exercise #3: Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $2x+3=17$ and $x=7$

$2(7)+3=17$
$14+3=17$ yes
$17=17$

(b) $\frac{x-20}{5}=-4$ and $x=10$

$\frac{10-20}{5}=-4$ $\frac{-10}{5}=-4$
$-2\neq -4$
no

(c) $2(x+5)=6(x-1)$ and $x=4$

$2(4+5)=6(4-1)$
$2(9)=6(4-1)$
$18=6(3)$ yes
$18=18$

(d) $x^2-1=2x+2$ and $x=-1$

$(-1)^2-1=2(-1)+2$
$1-1=-2+2$
$0=0$ yes

(e) $\frac{3(x+2)}{4}-1=5$ and $x=2$

$\frac{3(2+2)}{4}-1=5$
$\frac{3(4)}{4}-1=5$ $2\neq 5$
$3-1=5$ no

(f) $\frac{5}{4}x-1=-\frac{1}{2}x+9$ and $x=8$

$\frac{24}{4}-1=\frac{-8}{2}+9$ yes
$6-1=-4+9$
$5=5$

Sir, this is no excuse land. If you solve an equation, you should always be able to check to see if your solution is correct. Sometimes, mistakes happen, and it is good to be able to spot them.

Exercise #4: Kirk was checking to see if $x=7$ was a solution to the equation $4x-3=2x+11$. He concluded that it was not a solution based on the following work. Was he correct?

$4x-3=2x+11$

$4\cdot 7-3=2\cdot 7+11$

$4\cdot 4=2\cdot 18$

$16=36$ No!

$28-3=14+11$

$25=25$ ✓