Name: KEY   Date: ______

JUSTIFYING THE STEPS IN SOLVING AN EQUATION (EVEN STRANGE ONES)

Now that we have reviewed how to solve linear equations involving variables on both sides, it is time to take it to another level. The Common Core asks us not only to know the how but also the why. Generally, we justify the steps we take in solving linear equations using the commutative, associative, and distributive properties of real numbers along with the following two **properties of equality**.

> **PROPERTIES OF EQUALITY**
>
> (1) **ADDITIVE PROPERTY OF EQUALITY:** If $a=b$ then $a+c=b+c$ (you can add or subtract the same quantity from both sides and retain the equality).
>
> (2) **MULTIPLICATIVE PROPERTY OF EQUALITY:** If $a=b$ then $c\cdot a=c\cdot b$ (you can multiply or divide by the same quantity on both sides and retain the equality).

Exercise #1: Consider the equation $2x+9=21$. The steps in solving the equation are shown below. Justify each step.

Step 1: $2x+9=21$
$\quad -9 \;\; -9$ — Justification: additive property

Step 2: $\frac{2x}{2}=\frac{12}{2}$ — Justification: multiplicative property

$x=6$

O.k. That was a reasonably simple two-step equation. Now, let's go for the full experience.

Exercise #2: Consider the equation $3(x+2)-2(x+7)=4x+7$. As in the last problem, each step of the solution is shown. Justify each with either a property of equality or a property of real numbers.

Step 1: $3x+6-2x-14=4x+7$ — Justification: distribute

Step 2: $3x+-2x+6+-14=4x+7$ — Justification: Commutative property

Step 3: $\underline{3x-2x}+\underline{6-14}=4x+7$
$x-8=4x+7$ — Justification: Combine like terms

Step 4: $x-8=4x+7$
$-x \qquad -x$ — Justification: additive property

Step 5: $-8=3x+7$
$-7 \qquad\quad -7$ — Justification: additive property

Step 6: $\frac{-15}{3}=\frac{3x}{3}$ — Justification: multiplicative property

$x=-5$

Strange things can sometimes happen when you solve an equation. Even if every step is justified, results can turn out confusing.

Exercise #3: Consider the equation $5x-3(x+1)=2(x+4)$.

(a) Fill in this missing justifications in the solution of this equation below.

Step #1: $5x-3x-3=2x+8$ — Justification: The Distributive Property

Step #2: $2x-3=2x+8$ — Justification: Combine like terms

Step #3: $2x-3=2x+8$
$-2x \qquad -2x$ — Justification: additive property

Step #4: $-3=8$ — Justification: additive property

?

(b) The final line of this set of manipulations is a very strange statement: $-3=8$ Is this a true statement? Could any value of x make it a true statement?

no, there's no x in it so it can't

(c) What do you think this tells you about the solutions to this equation (i.e. the values of x that make it true)?

no solutions

Exercise #4: Consider the equation $7x+2(x+5)=9x+10$.

(a) Show that $x=-5$ and $x=2$ are both solutions to this equation.

-5: $7(-5)+2(-5+5)=9(-5)+10$
$-35+2(0)=-45+10$
$-35+0=-45+10$
$-35=-35$ ✓

2: $7(2)+2(2+5)=9(2)+10$
$14+2(7)=18+10$
$14+14=28$
$28=28$ ✓

(b) Solve this equation by manipulating each side of the equation as we did above. What does its final "strange" result tell you?

$7x+2(x+5)=9x+10$
$7x+2x+10=9x+10$
$9x+10=9x+10$
$-9x \qquad -9x$
$10=10$

always true, regardless of what x is

(c) Test your conclusion in (b) by picking a random integer (or really any number) and showing that it is a solution to the equation.

$x=3.2$

$7(3.2)+2(3.2+5)=9(3.2)+10$
$22.4+2(8.2)=28.8+10$
$22.4+16.4=38.8$
$38.8=38.8$ ✓

infinitely many solutions