

Name: KEY

Date: _____

INTRODUCTION TO POLYNOMIALS



The way we write numbers in our systems is interesting because with only 10 digits, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, we are able to write whole numbers as large as we would like. This is because what we really are doing is counting how many powers of 10 that we have.

Exercise #1: Write each of the following numbers as a sum of multiples of powers of 10. The first is done as an example.

$$\begin{aligned} \text{(a)} \quad 563 &= 500 + 60 + 3 \\ &= 5 \cdot 100 + 6 \cdot 10 + 3 \\ &= 5 \cdot 10^2 + 6 \cdot 10 + 3 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 274 &= 200 + 70 + 4 \\ &= 2 \cdot 100 + 7 \cdot 10 + 4 \\ &= 2 \cdot 10^2 + 7 \cdot 10 + 4 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 3,842 &= 3000 + 800 + 40 + 2 \\ &= 3 \cdot 1000 + 8 \cdot 100 + 4 \cdot 10 + 2 \\ &= 3 \cdot 10^3 + 8 \cdot 10^2 + 4 \cdot 10 + 2 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 5,081 &= 5000 + 80 + 1 \\ &= 5 \cdot 1000 + 8 \cdot 10 + 1 \\ &= 5 \cdot 10^3 + 8 \cdot 10 + 1 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 21,478 &= 20000 + 1000 + 400 + 70 + 8 \\ &= 2 \cdot 10000 + 1 \cdot 1000 + 4 \cdot 100 + 7 \cdot 10 + 8 \\ &= 2 \cdot 10^4 + 1 \cdot 10^3 + 4 \cdot 10^2 + 7 \cdot 10 + 8 \end{aligned}$$

We can now use algebra to replace the base of 10 with a generic base of x (or whatever variable you like).

Exercise #2: Consider the number 63,735.

(a) As in #1, write this number as the sum of multiples of powers of 10.

$$\begin{aligned} &60000 + 3000 + 700 + 30 + 5 \\ &6 \cdot 10000 + 3 \cdot 1000 + 7 \cdot 100 + 3 \cdot 10 + 5 \\ &6 \cdot 10^4 + 3 \cdot 10^3 + 7 \cdot 10^2 + 3 \cdot 10 + 5 \end{aligned}$$

(b) If $x=10$, write this number in terms of an equivalent expression involving x .

$$6x^4 + 3x^3 + 7x^2 + 3x + 5$$

The base of a polynomial certainly doesn't have to be 10. But, all polynomials have a form similar to your answer in letter (b). Let's define them a little more definitively.

POLYNOMIAL EXPRESSIONS

Any expression of the form: $ax^n + bx^{n-1} + cx^{n-2} + \dots + \text{constant}$, where the exponents, $n, n-1, n-2$, etcetera are all positive integers. Note that not all powers need to be presents because the coefficients, i.e. a, b, c , etcetera can be zero.

Exercise #3: Of the expressions shown below, circle all of them that represent polynomials. Discuss why the ones that aren't polynomials fail the definition above.

$$4x^2 + 8x + 1$$

$$9x^2 + 2x + \frac{1}{x}$$

$$3^x + 3^x + 4^x$$

$$2x^2 + 5x^3 - x + 8$$

X

X



It is often important to place polynomials in their standard form. The standard form of a polynomial is simply achieved by writing it as an equivalent expression where the powers on the variables always descend.

Exercise #4: Write each of the following polynomials in standard form.

(a) $3x^2 + 5x^3 + 7 - 8x$

$$5x^3 + 3x^2 - 8x + 7$$

(b) $9x^4 + 2x - x^2 + 1$

$$9x^4 - x^2 + 2x + 1$$

(c) $3 - 2x - 5x^2$

$$-5x^2 - 2x + 3$$

Polynomials are simply abstract representations of numbers that we see every day and they behave like these numbers as well. Let's look at adding polynomials together.

Exercise #5: Consider the numbers 523 and 271.

(a) Write each as the sum of multiples of powers of 10 as previously done.

$$5 \cdot 10^2 + 2 \cdot 10 + 3$$

$$2 \cdot 10^2 + 7 \cdot 10 + 1$$

(b) Add these numbers by adding each individual power of 10.

$$7 \cdot 10^2 + 9 \cdot 10 + 4$$

$$794$$

(c) Use this idea to add: $5x^2 + 2x + 3$

$$\begin{array}{r} 5x^2 + 2x + 3 \\ + 2x^2 + 7x + 1 \\ \hline 7x^2 + 9x + 4 \end{array}$$

(d) Find the sum of the polynomials $-4x^2 + 9x - 3$ and $7x^2 - 5x + 4$.

$$\begin{array}{r} -4x^2 + 9x - 3 \\ + 7x^2 - 5x + 4 \\ \hline 3x^2 + 4x + 1 \end{array}$$

Finding sums of polynomials is fairly easy. Subtracting them, though, can lead to a lot of errors.

Exercise #6: Find each of the following differences. Be careful and rewrite as an equivalent addition problem if necessary.

(a) $6x^2 + 5x + 3$

$$\begin{array}{r} 6x^2 + 5x + 3 \\ + 2x^2 + 4x + 7 \\ \hline 4x^2 + 9x + 4 \end{array}$$

(b) $(4x^2 - 2x + 7) - (x^2 + 4x - 1)$

$$6x^2 - 3x + 10$$

Exercise #7: For each of the following, write an equivalent polynomial in simplest standard form.

(a) $6x^2 + 2x - 3 - x^2 + 4x - 1$

$$5x^2 + 6x - 4$$

(b) $6x^2 + 2x - 3 - (x^2 + 4x - 1)$

$$6x^2 + 2x - 3 - x^2 - 4x + 1$$

$$5x^2 - 2x - 2$$

