2E

Name: KEY   Date: ____________

COMPLETING THE SQUARE

The turning point of a parabola and its general shape are relatively easy to determine if the quadratic function is written in its shifted or vertex form. Review this in the first exercise.

vertex form: $y = a(x-h)^2 + k$

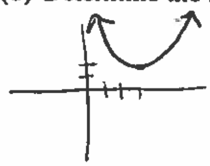
vertex: (h, k)

Exercise #1: Given the function $y = (x-3)^2 + 2$ do the following.

(a) Give the coordinates of the turning point.

$(3, 2)$

(b) Determine the range by drawing a rough sketch.



range: $y \geq 2$

The question then is how we take a quadratic of the form $y = ax^2 + bx + c$ and put it into its shifted form. This procedure is known as **Completing the Square.** But, it needs some additional review.

Exercise #2: Write each of the following as an equivalent trinomial.

(a) $(x+5)^2$ $(x+5)(x+5)$

$x^2 + 5x + 5x + 25$

$x^2 + 10x + 25$

(b) $(x-1)^2$ $(x-1)(x-1)$

$x^2 - x - x + 1$

$x^2 - 2x + 1$

(c) $(x+4)^2$ $(x+4)(x+4)$

$x^2 + 4x + 4x + 16$

$x^2 + 8x + 16$

Exercise #3: Given each trinomial in Exercise #2 of the form $x^2 + bx + c$, what is true about the relationship between the value of b and the value of c? Illustrate.

$c = \left(\frac{b}{2}\right)^2$

Exercise #4: Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.

(a) $x^2 + 20x + 100$

$(x+10)^2$

(b) $x^2 - 6x + 9$

$(x-3)^2$

(c) $x^2 + 2x + 1$

$(x+1)^2$

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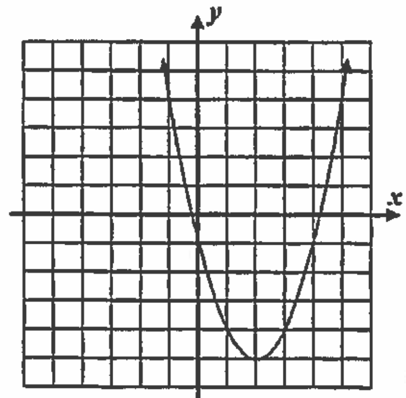
We are finally ready to learn the method of **Completing the Square**. This method has many uses, but the one we will work with today is to manipulate equations of quadratics from their **standard form** to their **vertex form**.



Exercise **#5:** The quadratic $y = x^2 - 4x - 1$ is shown graphed below.

(a) Consider only the binomial $x^2 - 4x$. What would you need to add on to it to create a perfect square trinomial? (See Exercise #3).

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(b) In order to add zero to the binomial $x^2 - 4x$, what should we subtract to offset adding 4 to make it a perfect square?

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(c) Use the Method of Competing the Square now to rewrite the trinomial $x^2 - 4x - 1$ in an equivalent, shifted form. According to this form, what are the coordinates of the vertex? Verify by examining the graph.

$x^2 - 4x + 4 - 4 - 1$

$(x-2)^2 - 5$

This procedure is what is known as an **algorithm**. In other words, we follow a recipe. Here it is:

COMPLETING THE SQUARE

For the quadratic $y = x^2 + bx + c$ (note that $a = 1$).

1. Find half of the value of *b*, i.e. $\frac{b}{2}$ 2. Square it, i.e. $\left(\frac{b}{2}\right)^2$ 3. Add and subtract it

There is nothing like practice on these.

Exercise **#6:** Write each quadratic in vertex form by Completing the Square. Then, identify the quadratic's turning point. The last two problems will involve fractions. Stick with it!

(a) $y = x^2 + 6x - 2$

$\frac{6}{2} = 3 \quad 3^2 = 9$

$x^2 + 6x + 9 - 9 - 2$

$(x+3)^2 - 11$

(b) $y = x^2 - 2x + 11$

$\frac{-2}{2} = -1 \quad (-1)^2 = 1$

$x^2 - 2x + 1 - 1 + 11$

$(x-1)^2 + 10$

(c) $y = x^2 - 10x + 27$

$\frac{-10}{2} = -5 \quad (-5)^2 = 25$

$x^2 - 10x + 25 - 25 + 27$

$(x-5)^2 + 2$

(d) $y = x^2 + 8x$

$\frac{8}{2} = 4 \quad 4^2 = 16$

$x^2 + 8x + 16 - 16$

$(x+4)^2 - 16$

(e) $y = x^2 + 5x + 4$

$\frac{5}{2} = 2.5 \quad (2.5)^2 = 6.25$

$x^2 + 5x + 6.25 - 6.25 + 4$

$(x+2.5)^2 - 2.25$

(f) $y = x^2 - 9x - 2$

$\frac{-9}{2} = -4.5 \quad (-4.5)^2 = 20.25$

$x^2 - 9x + 20.25 - 20.25 - 2$

$(x-4.5)^2 - 22.25$