Factor Polynomials Completely

Goal Factor polynomials completely

Try to group terms and look for common factors.

Example 1 Factor out common binomial

a. Factor the expression $5x^2(x-3)-4(x-3)$
Solution $5x^2(x-3)-4(x-3) = (5x^2-4)(x-3)$

b. Factor the expression $3x^2(x-3)+24(3-x)$
Solution $3x^2(x-3) + 24(3-x) = 3x^2(x-3) - 24(x-3) = (3x^2-24)(x-3)$

PROBLEMS

1. $11x(x+7)+3(x+7)$

$(x+7)(11x+3)$

2. $x^2(x+1)-4(x+1)$

$(x+1)(x^2-4)$

3. $x^2(14x+9)-3(14x+9)$

$(14x+9)(x^2-3)$

4. $2x(x-7)+3(\underset{(x-7)}{7-x})$

$(x-7)(2x-3)$

5. $7y(6-y)+3(\underset{(6-y)}{y-6})$

$(7y-3)(6-y)$

6. $11x(x-7)+3(\underset{(x-7)}{7-x})$

$(x-7)(11x+3)$

Example 2 Factor by grouping

a. Factor the expression $x^3 + 7x^2 - 2x - 14$

Solution $x^3 + 7x^2 - 2x - 14 = (x^3 + 7x^2) + (-2x-14)$
$= x^2(x+7) - 2(x+7) = (x^2-2)(x+7)$

PROBLEMS

7. $x^3 + 6x^2 + 5x + 30$

$x^2(x+6)+5(x+6)$

$(x^2+5)(x+6)$

8. $9x^2 + 9x - 7x - 7$

$9x(x+1) - 7(x+1)$

$(x+1)(9x-7)$

9. $2x^3 - 6x^2 + 4x - 12$

$2x^2(x-3) + 4(x-3)$

$(2x^2+4)(x-3)$

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Example 3   Factor the polynomial completely

a. Factor the expression $5x^3 - 55x^2 + 90x$

Solution
$$5x^3 - 55x^2 + 90x = 5x(x^2 - 11x + 18)$$
$$= 5x(x-2)(x-9)$$

b. Factor the expression $48x^4 - 3x^2$

Solution
$$48x^4 - 3x^2 = 3x^2(16x^2 - 1)$$
$$= 3x^2(4x+1)(4x-1)$$

PROBLEMS

10. $2x^3 - 16x^2 + 32x$ 16, -4 -4
$2x(x^2 - 8x + 16)$
$2x(x-4)(x-4)$
$2x(x-4)^2$

11. $3x^3 + 18x^2 + 24x$ 8, 4 2
$3x(x^2 + 6x + 8)$
$3x(x+4)(x+2)$

12. $2x^3 - 4x^2 - 30x$ -15, -5 3
$2x(x^2 - 2x - 15)$
$2x(x-5)(x-3)$

13. $5x^3 - 20x$
$5x(x^2 - 4)$
$5x(x-2)(x+2)$

14. $7x^3 - 63x$
$7x(x^2 - 9)$
$7x(x+3)(x-3)$

15. $3x^3 - 48x$
$3x(x^2 - 16)$
$3x(x+4)(x-4)$

Example 4   Solve a polynomial equation

Solve the equation $4x^3 + 4x^2 - 24x = 0$

Solution
$$4x^3 + 4x^2 - 24x = 0$$
$$4x(x^2 + x - 6) = 0$$
$$4x(x+3)(x-2) = 0$$

$$x = 0 \quad x = -3 \quad x = 2$$

The solutions of the equation are 0, -3, and 2.

PROBLEMS   Solve the polynomial equation:

16. $2x^3 + 18x^2 + 40x = 0$
$2x(x^2 + 9x + 20) = 0$
$2x(x+4)(x+5) = 0$
$$\boxed{X = 0, -4, -5}$$

17. $3x^3 + 9x^2 + 6x = 0$
$3x(x^2 + 3x + 2) = 0$
$3x(x+2)(x+1) = 0$
$$\boxed{X = 0, -2, -1}$$

18. $4x^3 + 12x^2 - 72x = 0$
$4x(x^2 + 3x - 18) = 0$
$4x(x+6)(x-3) = 0$
$$\boxed{X = 0, -6, 3}$$