Unit 1: Conversions and Simplifying Radicals

UNIT CONVERSIONS

Units are amazingly important in mathematics, science, and engineering. They are how we decide on what constitutes the number 1 (i.e. 1 gallon, 1 pound, etcetera). We often need to **convert** from one unit to another in practical problems. In this situation we can almost always use proportional reasoning to do the job.

***Exercise* #1:** John has traveled a total of 4.5 miles. If there are 5,280 feet in each mile, how many feet did John travel? Set up and solve a proportion for this problem. Also, do the problem by multiplying by a ratio.

***Exercise* #2:** If there are exactly 2.54 centimeters in each inch, how many centimeters are in one foot? Show the work that leads to your answer.

Sometimes it is helpful to be able to convert so that a rate makes more sense. Take a look at the next problem.

***Exercise* #3:** A bathtub contains 14.5 cubic feet of water. If water drains out of the bathtub at a rate of 4 gallons per minute, then how long will it take, to the nearest minute, to drain the bathtub? There are 7.5 gallons of water per cubic foot. Show the work that leads to your answer.

***Exercise* #4:** The mile and the kilometer are relatively close in size. Can you convert 1 mile into an equivalent in kilometers? Here's what I'll give you. There are 2.54 centimeters in an inch, 5,280 feet in a mile, 100 centimeters in a meter, and 1000 meters in a kilometer. All else you should be able to do for yourself. Round your answer to the nearest tenth of a kilometer. This takes quite a string of multiplications, but you can do it!

We can also convert the ratio of two quantities, or rates, into different units if need be.

***Exercise* #5:** One interesting conversion is from a speed expressed in feet per second to a speed in miles per hour. We sometimes think better in miles per hour because that is how the speeds of our cars are measured.

(a) Convert a speed of 45 miles per hour into feet per second given that there are 5,280 feet in a mile.

(b) The current fastest human is Usain Bolt, from Jamaica. In 2009, Usain ran 100 meters in a blazing 32.2 feet per second average speed. How does this compare to a typical car driving speed?

***Exercise* #6:** A local factory has to add a liquid ingredient to make their product at a rate of 13 quarts every 5 minutes. How many gallons per hour of the ingredient do they need to add? Show the work that leads to your answer.

***Exercise* #7:** A tractor can plant a field at a rate of 2.5 acres per 5 minutes. If a mammoth farm measuring 4 square miles needs planting, how long will it take in hours to plant the field? There are 640 acres in a square mile. Determine your answer to the nearest hour. If the tractor can run 8 hours a day, what is minimum number of days it will take to plant the farm?

Name: ____________________ Date: __________

Unit Conversion Worksheet

Conversions

1 hour = 3600 seconds
1 meter = 3.28 feet
1 kg = 2.2 lbs
1 m/s = 2.2 miles/hour
1 mile = 5280 feet
1 km = 0.62 miles
1 lb = 0.45 kg
1 foot = 12 inches
1 yard = 3 feet
1 light second = 300,000,000 meters
1 quart = 0.946 liters
1 inch = 2.54 cm = 25.4 mm

Convert the following quantities.

1. 565,900 seconds into days

$$\frac{565{,}900\,\text{sec}}{1} \cdot \frac{\text{min}}{\text{sec}} \cdot \frac{\text{hr}}{\text{min}} \cdot \frac{\text{day}}{\text{hr}} =$$

2. 17 years into minutes

$$\frac{17\,\text{yr}}{1} \cdot \frac{\text{days}}{\text{yr}} \cdot \frac{\text{hrs}}{\text{days}} \cdot \frac{\text{min}}{\text{hr}} =$$

3. 43 miles into feet

$$\frac{43\,\text{miles}}{1} \cdot \frac{\text{ft}}{\text{miles}} =$$

4. 165 pounds into kilograms

$$\frac{165\,\text{lbs}}{1} \cdot \frac{\text{kg}}{\text{lbs}} =$$

5. 100 yards into meters

$$\frac{100\,\text{yd}}{1} \cdot \frac{\text{ft}}{\text{yd}} \cdot \frac{\text{m}}{\text{ft}} =$$

6. 22,647 inches into miles

$$\frac{22{,}647\,\text{in}}{1} \cdot \frac{\text{ft}}{\text{in}} \cdot \frac{\text{mi}}{\text{ft}} =$$

7. 2678 cm into feet

$$\frac{2678\,\text{cm}}{1} \cdot \frac{\text{in}}{\text{cm}} \cdot \frac{\text{ft}}{\text{in}} =$$

8 60 miles per hour into meters per second

9 130 meters per second into miles per hour

10 1100 feet per second into miles per hour

11 53 yards per hour into inches per week

12 721 lbs per week into kg per second

13 88 inches per second into miles per day

14 12080 gallons per week into liters per hour

15 27 miles per gallon into kilometers per liter

16 186,282 miles per second into meters per second

Name: ______________________________ Date: ________________

UNIT CONVERSIONS

APPLICATIONS

1. How many centimeters are there in 1 yard if there are 2.54 centimeters per inch? Show your work and express your answer without rounding.

2. How close are a meter and a yard? Convert 1 meter into yards by using the fact that there are 100 centimeters in a meter, 2.54 centimeters in an inch, 12 inches in a foot, and 3 feet in a yard. Round your answer to the nearest tenth of a yard.

3. If there are 1000 grams in a kilogram and 454 grams in a pound, how many pounds are there per kilogram? Round to the nearest tenth of a pound.

4. Water is flowing out of an artesian spring at a rate of 8 cubic feet per minute. How many minutes will it take for the water to fill up a 300 gallon tank. There are 7.5 gallons of water per cubic foot. Show or explain how you arrive at your answer.

5. A high school track athlete sprints 100 yards in 15 seconds.

(a) Determine the number of feet per second the runner is traveling at. Show your work.

(b) If there are 5280 feet in a mile and 3600 seconds in an hour, determine the runner's speed in miles per hour. Round to the nearest tenth.

6. A cafeteria is trying to scale a small pancake recipe up in order to feed a group of tourists. The recipe feeds 6 people and the cafeteria is trying to feed 75. The recipe calls for 4 cups of flour and $1\frac{1}{2}$ cups of milk and ½ cup of sugar (as well as some other minor ingredients such as baking powder).

(a) One 10 pound bag of flour contains 38 cups of flour. Will it be enough for this recipe? Justify.

(b) If one 10 pound bag of flour contains 38 cups of flour, how many pounds of flour will be needed for this recipe? Round to the nearest tenth of a pound.

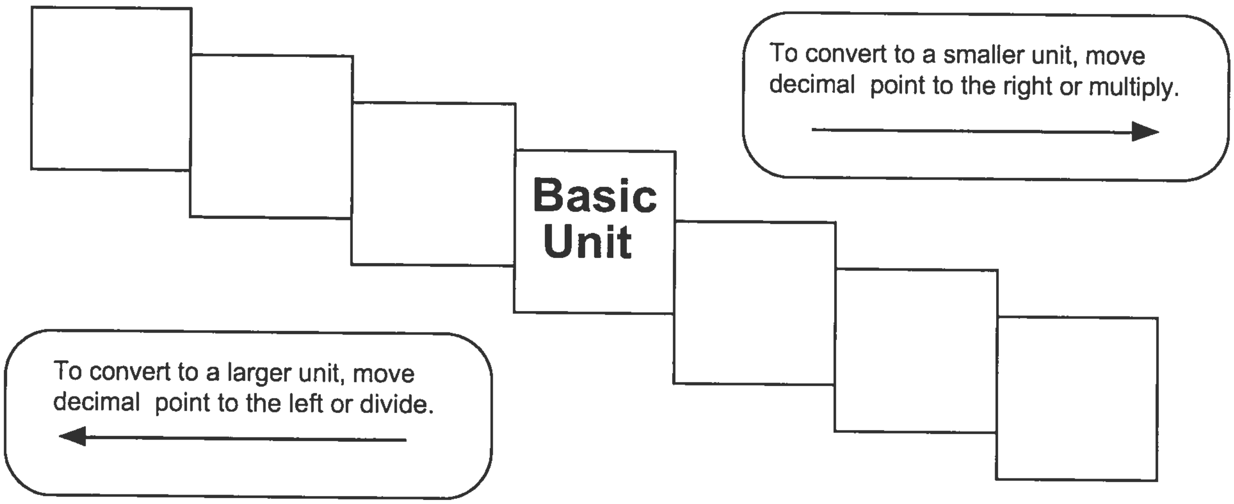
(c) If there are 4 cups in a quart and 4 quarts in a gallon, will we need more or less than a gallon of milk for this recipe?

(d) The cafeteria has a 1.5 kilogram bag of sugar. If a cup of sugar weighs 0.5 pounds and there are 2.2 pounds per kilogram, does the cafeteria have enough sugar to make this recipe?

(e) If the original recipe made 14 pancakes and the cafeteria plans to charge $.50 per pancake, how much money will they make if they sell all of the pancakes made for the 75 people?

Metric Mania
Conversion Practice

Name______________________________



Try these conversions, using the ladder method.

1000 mg = ________ g

1 L = ________ mL

160 cm = ________ mm

14 km = ________ m

109 g = ________ kg

250 m = ________ km

Compare using <, >, or =.

56 cm ◯ 6 m

7 g ◯ 698 mg

Metric Mania

Name____________________________

Conversion Challenge

Write the correct abbreviation for each metric unit.

1) Kilogram _____
2) Meter _____
3) Gram _____
4) Milliliter _____
5) Millimeter _____
6) Liter _____
7) Kilometer _____
8) Centimeter _____
9) Milligram _____

Try these conversions, using the ladder method.

1) 2000 mg = _______ g
2) 104 km = _______ m
3) 480 cm = _____ m
4) 5.6 kg = _____ g
5) 8 mm = _____ cm
6) 5 L = _______ mL
7) 198 g = _______ kg
8) 75 mL = _____ L
9) 50 cm = _____ m
10) 5.6 m = _____ cm
11) 16 cm = _______ mm
12) 2500 m = _______ km
13) 65 g = _____ mg
14) 6.3 cm = _____ mm
15) 120 mg = _____ g

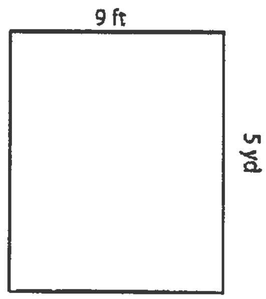
Compare using <, >, or =.

16) 63 cm ◯ 6 m
17) 5 g ◯ 508 mg
18) 1,500 mL ◯ 1.5 L
19) 536 cm ◯ 53.6 dm
20) 43 mg ◯ 5 g
21) 3.6 m ◯ 36 cm

Rectangle - Area

perimeter +
Find the area of each rectangle.

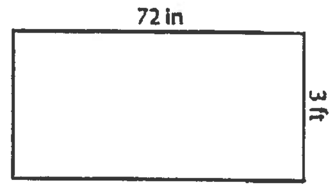
1)



Area = ________ yd^2

P = ______ yd.

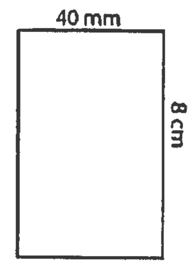
2)



Area = ________ ft^2

P = ______ ft

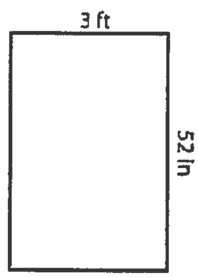
3)



Area = ________ cm^2

P = ______ cm

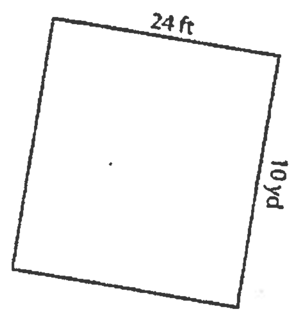
4)



Area = ________ in^2

P = ______ in

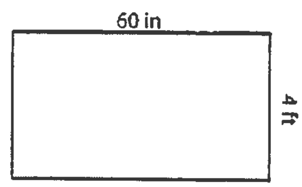
5)



Area = ________ yd^2

P = ______ yd

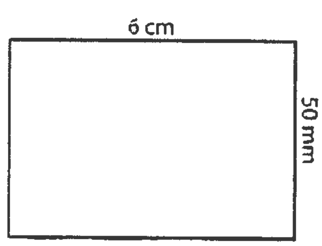
6)



Area = ________ ft^2

P = ______ ft

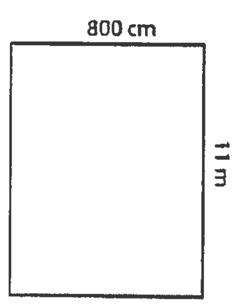
7)



Area = ________ cm^2

P = ______ cm

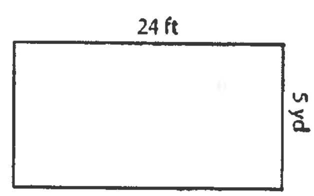
8)



Area = ________ m^2

P = ______ m

9)



Area = ________ ft^2

P = ______ yd

SQUARE ROOTS

Square roots, cube roots, and higher level roots are important mathematical tools because they are the **inverse operations** to the operations of **squaring and cubing**. In this unit we will study these operations, as well as numbers that come from using them. First, some basic review of what you've seen before.

***Exercise* #1:** Find the value of each of the following **principal square roots**. Write a reason for your answer in terms of a multiplication equation.

(a) $\sqrt{25}$ (b) $\sqrt{9}$ (c) $\sqrt{100}$

(d) $\sqrt{0}$ (e) $\sqrt{\frac{1}{4}}$ (f) $\sqrt{\frac{64}{9}}$

It is generally agreed upon that all **positive, real numbers** have two square roots, a positive one and a negative one. We simply designate which one we want by either including a negative sign or leaving it off.

***Exercise* #2:** Give all square roots of each of the following numbers.

(a) 4 (b) 36 (c) $\frac{1}{16}$

***Exercise* #3:** Given the function $f(x)=\sqrt{x+3}$, which of the following is the value of $f(46)$?

(1) 22 (3) 16

(2) 5 (4) 7

Square roots have an interesting property when it comes to multiplication. We will discover that property in the next exercise.

***Exercise* #4:** Find the value of each of the following products.

(a) $\sqrt{4}\cdot\sqrt{9}=$ (b) $\sqrt{4\cdot 9}=$

(c) $\sqrt{4}\cdot\sqrt{25}=$ (d) $\sqrt{4\cdot 25}=$

What you should notice in the last exercise is the following important property of square roots.

MULTIPLICATON PROPERTY OF SQUARE ROOTS

1. $\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$ likewise 2. $\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$

One obvious use for this is to multiply two "unfriendly" square roots to get a nice result.

***Exercise* #5:** Find the result of each of the following products.

(a) $\sqrt{2}\cdot\sqrt{8}=$ (b) $\sqrt{12}\cdot\sqrt{3}=$ (c) $\sqrt{20}\cdot\sqrt{5}=$

One less obvious use for the square root property above is in **simplifying square roots of non-perfect squares**. This is a fairly antiquated skill that is almost completely irrelevant to algebra, but it often arises on standardized tests and thus is a good skill to become fluent with.

***Exercise* #6:** To introduce **simplifying square roots**, let's do the following first.

(a) List out the first 10 perfect squares (starting with 1).

(b) Now consider $\sqrt{18}$. Which of these perfects squares is a factor (divides) of 18?

(c) Simplify the $\sqrt{18}$. This is known as writing it in **simplest radical form**.

The key to simplifying any square root is to find the **largest perfect square** that is a factor of the **radicand** (the number under the square root).

***Exercise* #7:** Write each of the following square roots in simplest radical form.

(a) $\sqrt{8}$ (b) $\sqrt{45}$ (c) $\sqrt{48}$

(d) $-\sqrt{75}$ (e) $\sqrt{72}$ (f) $-\sqrt{500}$

Name: ______________________________ Date: ______________

SQUARE ROOTS

FLUENCY

1. Simplify each of the following. Each will result in a rational number answer. You can check your work using your calculator, but should try to do all of them without it.

(a) $\sqrt{36}$ (b) $-\sqrt{4}$ (c) $\sqrt{121}$ (d) $\sqrt{\frac{1}{9}}$

(e) $-\sqrt{100}$ (f) $\sqrt{\frac{81}{36}}$ (g) $-\sqrt{\frac{1}{16}}$ (h) $-\sqrt{144}$

2. Find the final, simplified answer to each of the following by evaluating the square roots first. Show the steps that lead to your final answers.

(a) $\sqrt{9}+\sqrt{25}-\sqrt{64}$ (b) $5\sqrt{4}+2\sqrt{81}$

(c) $\frac{2\sqrt{25}+2}{3}$ (d) $\sqrt{\frac{1}{4}}\left(\sqrt{121}-\sqrt{9}\right)$

All of the square roots so far have been "nice." We will discuss what this means more in the next lesson. We can use the Multiplication Property to help simplify certain products of not-so-nice square roots.

3. Find each of the following products by first multiplying the **radicands** (the numbers under the square roots).

(a) $\sqrt{2}\cdot\sqrt{50}=$ (b) $\sqrt{3}\cdot\sqrt{12}=$ (c) $5\sqrt{6}\cdot\sqrt{24}=$

(d) $\sqrt{25}-\sqrt{2}\cdot\sqrt{8}=$ (e) $\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{18}}=$ (f) $\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{27}{4}}=$

4. Write each of the following in **simplest radical form**. Show the work that leads to your answer. The first exercise has been done to remind you of the procedure.

(a) $\sqrt{162} =$
$= \sqrt{81} \cdot \sqrt{2}$
$= 9\sqrt{2}$

(b) $\sqrt{20} =$

(c) $-\sqrt{90} =$

(d) $\sqrt{48} =$

(e) $-\sqrt{8} =$

(f) $\sqrt{300} =$

5. Write each of the following products in **simplest radical form**. The first is done as an example for you.

(a) $3\sqrt{12} =$
$= 3 \cdot \sqrt{4} \cdot \sqrt{3}$
$= 3 \cdot 2 \cdot \sqrt{3}$
$= 6\sqrt{3}$

(b) $4\sqrt{45} =$

(c) $\frac{1}{2}\sqrt{32} =$

(d) $-2\sqrt{288} =$

(e) $\frac{\sqrt{108}}{3} =$

(f) $\frac{-\sqrt{320}}{16} =$

REASONING

It is critical to understand that when we "simplify" a square root or perform any calculation using them, we are always finding **equivalent numerical expressions**. Let's make sure we see that in the final exercise.

6. Consider $\sqrt{28}$.

(a) Use your calculator to determine its value. Round to the nearest *hundredth*.

(b) Write $\sqrt{28}$ in **simplest radical form.**

(c) Use your calculator to find the value of the product from part (b). How does it compare to your answer from (a)?

(d) Do the same comparison for $\sqrt{80}$.

Decimal Approximation: $\sqrt{80} =$

Simplified and then Approximated: $\sqrt{80} =$

Algebra 1

Simplifying Radicals Practice

Simplify.

1) $\sqrt{36}$

2) $\sqrt{50}$

3) $\sqrt{8}$

4) $\sqrt{75}$

5) $\sqrt{12}$

6) $\sqrt{16}$

7) $\sqrt{100}$

8) $\sqrt{45}$

9) $\sqrt{512}$

10) $\sqrt{144}$

11) $\sqrt{256}$

12) $\sqrt{32}$

13) $\sqrt{448}$

14) $\sqrt{54}$

15) $\sqrt{180}$

16) $\sqrt{125}$

17) $\sqrt{20}$

18) $\sqrt{45}$

19) $\sqrt{8}$

20) $\sqrt{147}$

21) $\sqrt{343}$

22) $\sqrt{192}$

23) $\sqrt{16}$

24) $\sqrt{32}$

25) $\sqrt{18}$

26) $\sqrt{12}$

27) $\sqrt{54}$

28) $\sqrt{45}$

29) $\sqrt{80}$

30) $\sqrt{50}$

31) $\sqrt{48}$

32) $\sqrt{72}$

IRRATIONAL NUMBERS

The set of real numbers is made up of two distinctly different numbers. Those that are **rational** and those that are **irrational**. Their technical definitions are given below.

RATIONAL AND IRRATIONAL NUMBERS

1. A **rational rumber** is any number that can be written as the **ratio** of **two integers**. Such numbers include $\frac{3}{4}$, $\frac{-7}{3}$, and $\frac{5}{1}$. These numbers have **terminating** or **repeating decimals**.
2. An **irrational number** is any number that is **not rational**. So, ones that cannot be written as the ratio of two integers. These numbers have **nonterminating and nonrepeating decimal representations.**

***Exercise* #1:** Let's consider a number that is rational and one that is irrational (**not rational**). Consider the rational number $\frac{2}{3}$ and the irrational number $\sqrt{\frac{1}{2}}$. Both of these numbers are less than 1.

(a) Draw a pictorial representation of $\frac{2}{3}$ of the rectangle shown below.

(b) Using your calculator, give the decimal representation of the number $\frac{2}{3}$. Notice that it has a repeating decimal pattern.

(c) Write out all of the decimal places that your calculator gives you for $\sqrt{\frac{1}{2}}$. Notice that it does not have a repeating decimal pattern.

(d) Why could you not draw a pictorial representation of $\sqrt{\frac{1}{2}}$ that way you do for $\frac{2}{3}$?

Irrational numbers are necessary for a variety of reasons, but they are somewhat of a mystery. In essence they are a number that can never be found by **subdividing** an **integer quantity** into a **whole number** of **parts** and then taking an **integer number** of those parts. There are many, many types of irrational numbers, but **square roots of non-perfect squares** are **always irrational**. The proof of this is beyond the scope of this course.

***Exercise* #2:** Write out every decimal your calculator gives you for these **irrational numbers** and notice that they never repeat.

(a) $\sqrt{2} =$

(b) $\sqrt{10} =$

(c) $\sqrt{23} =$

Rational and irrational numbers often mix, as when we simplify the square root of a non-perfect square.

***Exercise* #3:** Consider the **irrational** number $\sqrt{28}$.

(a) Without using your calculator, between what two consecutive integers will this number lie? Why?

(b) Using your calculator, write out all decimals for $\sqrt{28}$.

(c) Write $\sqrt{28}$ in simplest radical form.

(d) Write out the decimal representation for your answer from (c). Notice it is the same as (b).

O.k. So, it appears that a **non-zero rational number times an irrational number** (see letter (c) above) results in an **irrational number** (see letter (d) above). We should also investigate what happens when we add rational numbers to irrational numbers (and subtract them).

***Exercise* #4:** For each of the following addition or subtraction problems, a rational number has been added to an irrational number. Write out the decimal representation that your calculator gives you and classify the result as rational (if it has a repeating decimal) or irrational (if it doesn't).

(a) $\frac{1}{2}+\sqrt{2}$

(b) $\frac{4}{3}+\sqrt{10}$

(c) $7-\sqrt{8}$

***Exercise* #5:** Fill in the following statement about the sum or rational and irrational numbers.

When a **rational number** is added to an **irrational number** the result is always ____________________.

***Exercise* #6:** Which of the following is an irrational number? If necessary, play around with your calculator to see if the decimal representation does not repeat. **Don't be fooled by the square roots.**

(1) $\sqrt{25}$

(3) $\frac{7}{2}$

(2) $4-\sqrt{9}$

(4) $3+\sqrt{6}$

Name: ______________________________ Date: ______________

IRRATIONAL NUMBERS

FLUENCY

1. For each of the following rational numbers, use your calculator to write out either the terminating decimal or the repeating decimal patterns.

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$ (c) $\frac{5}{8}$ (d) $\frac{5}{6}$

(e) $\sqrt{\frac{25}{4}}$ (f) $\sqrt{\frac{1}{100}}$ (g) $\sqrt{\frac{4}{9}}$ (h) $\sqrt{\frac{2}{32}}$

2. One of the most famous **irrational numbers** is the number pi, π, which is essential in calculating the circumference and area of a circle.

(a) Use your calculator to write out all of the decimals your calculator gives you for π. Notice no repeating pattern.

(b) Historically the rational number $\frac{22}{7}$ has been used to **approximate** the value of π. Use your calculator to write out all of the decimals for this rational number and compare it to (a).

3. For each of the following irrational numbers, do two things: (1) write the square root in simplest radical form and then (2) use your calculator to write out the decimal representation.

(a) $\sqrt{32}$ (b) $\sqrt{98}$ (c) $\sqrt{75}$

(d) $\sqrt{500}$ (e) $\sqrt{80}$ (f) $\sqrt{117}$

REASONING

Types of numbers mix and match in various ways. The last exercise shows us a trend that we explored during the lesson.

4. Fill in the statement below based on the last exercise with one of the words below the blank.

   The product of a (non-zero) rational number and an irrational number results in a(n) ____________ number.
   rational irrational

Now we will explore other patterns in the following exercises.

5. Let's explore the **product** of **two irrational numbers** to see if it is **always irrational, sometimes irrational, sometimes rational,** or **always rational.** Find each product below using your calculator (be careful as you put it in) and write out all decimals. Then, classify as either rational or irrational.

   (a) $\sqrt{5}\cdot\sqrt{3}=$ ____________________ Rational or irrational?

   (b) $\sqrt{8}\cdot\sqrt{18}=$ ____________________ Rational or irrational?

   (c) $\sqrt{7}\cdot\sqrt{11}=$ ____________________ Rational or irrational?

   (d) $\sqrt{11}\cdot\sqrt{11}=$ ____________________ Rational or irrational?

6. Based on #5, classify the following statement as true or false:

   Statement: The product of two irrational number is always irrational. True or False

7. Let's explore adding rational numbers. Using what you learned about in middle school, add each of the following pairs of rational numbers by first finding a **common denominator** then combine. Then, determine their repeating or terminating decimal.

   (a) $\frac{1}{2}+\frac{2}{3}=$ (b) $\frac{3}{4}+\frac{1}{2}=$ (c) $\frac{3}{8}+\frac{5}{12}=$

   (d) Classify the following statement as true or false:

   Statement: The sum of two rational numbers is always rational. True or False

8. Finally, what happens when we add a rational and an irrational number (we explored this in Exercises #4 through #6 in the lesson). Fill in the blank below from what you learned in class.

   The sum of a rational number with an irrational number will always give a(n) ______________ number.
   rational irrational