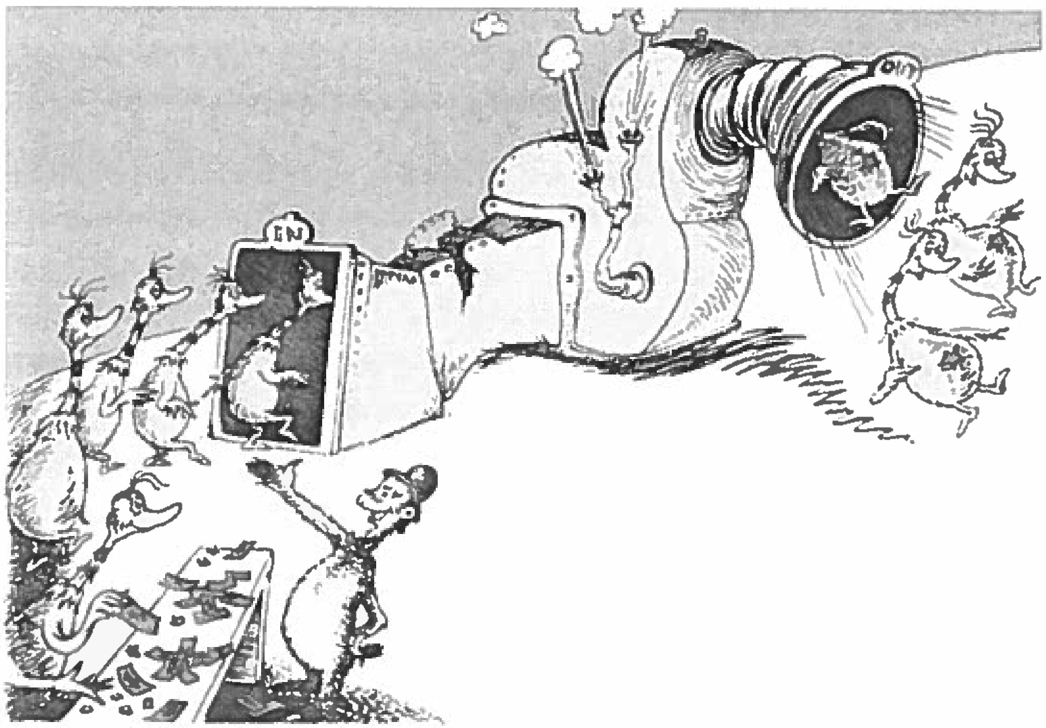
Unit $3\frac{1}{2}$: Functions



SYLVESTER McMONKEY McBEAN'S STAR-BELLY SNEETCH MACHINE — DR. SEUSS

$$f(x) = \frac{2x+1}{2} \longrightarrow f(3) = \frac{2(3)+1}{2}$$

$$f(3) = \frac{6+1}{2}$$

$$f(3) = 3\frac{1}{2}$$

Dr. Seuss "The Sneetches" Function Machine

How much did it cost to have a star belly? ___________

Write and equation in which x represents the number of sneetches and y represents the total cost. __________

Number of Sneetches Independent Variable (x)		Cost ($) Dependent Variable (y)
1		
2		
3		
4		
5		
6		

What type of sneetch do you like more? ______________

Why? ____

2

How much does it cost to have a star removed? __________

Write an equation in which x represents the number of sneetches and y represents the total cost.

Number of Sneetches Independent Variable (x)		Cost ($) Dependent Variable (y)
1		
2		
3		
4		
5		
6		

Define what a function is.

Function:

Name: ______________________________ Date: _______________

Introduction to Functions

The concept of the **function** ranks near the top of the list in terms of important Algebra concepts. Almost all of higher-level mathematical modeling is based on the concept. Like most important ideas in math, it is relatively simple:

> **The Definition of a Function**
>
> A **function** is a clearly defined **rule** that converts an **input** into **at most one output**. These rules often come in the form of: (1) equations, (2) graphs, (3) tables, and (4) verbal descriptions.

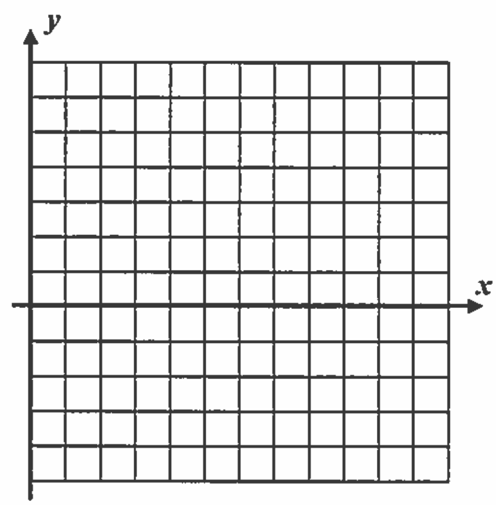
***Exercise* #1:** Consider the function rule: multiply the input by two and then subtract one to get the output.

(a) Fill in the table below for inputs and outputs. Inputs are often designated by x and outputs by y.

Input x	Calculation	Output y
0		
1		
2		
3		

(b) Write an equation that gives this rule in symbolic form.

(c) Graph the function rule on the graph paper shown below. Use your table in (a) to help.



***Exercise* #2:** In the function rule from #1, what input would be needed to produce an output of 17? Why is it harder to find an input when you have an output than finding an output when you have an input?

***Exercise* #3:** A function rule takes an input, n, and converts it into an output, y, by increasing one half of the input by 10. Determine the output for this rule when the input is 50 and then write an equation for the rule.

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Exercise #4: Function rules do not always have to be numerical in nature, they simply have to return a single output for a given input. The table below gives a rule that takes as an input a neighborhood child and gives as an output the month he or she was born in.

Child	Birth Month
Max	January
Evin	April
Zeke	May
Rosie	February
Niko	May

(a) Why can we consider this rule a function?

(b) What is the output when the input is Rosie?

(c) Find all inputs that give an output of May. Why does this *not* violate the definition of a function even though there are two answers?

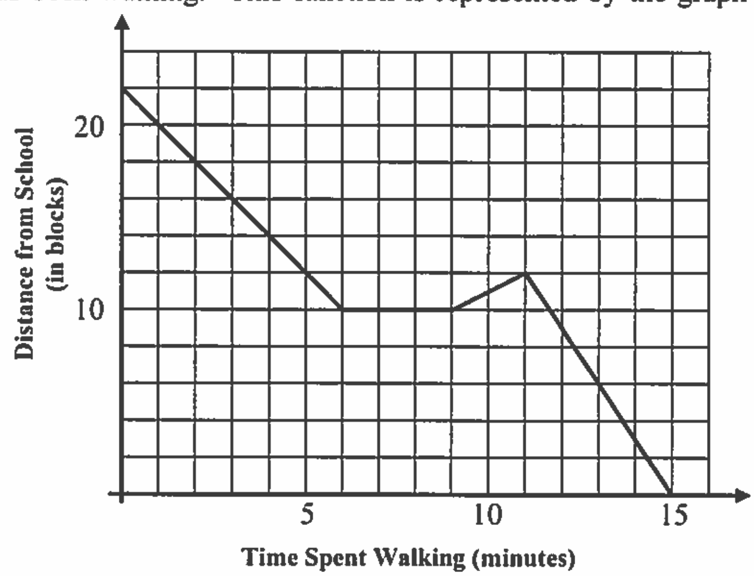
Functions are useful because they can often be used to **model** things that are happening in the real world. The next exercises illustrates a function given only in graphical form.

Exercise #5: Charlene heads out to school by foot on a fine spring day. Her distance from school, in blocks, is given as a function of the time, in minutes, she has been walking. This function is represented by the graph given below.

(a) How far does Charlene start off from school?

(b) What is her distance from school after she has been walking for 4 minutes?

(c) After walking for six minutes, Charlene stops to look for her subway pass. How long does she stop for?



(d) Charlene then walks to a subway station before heading to school on the subway (a local). How many blocks did she walk to the subway?

(e) How long did it take for her to get to school once she got on the train?

Name: ______________________________ Date: ________________

INTRODUCTION TO FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Decide whether each of the following relations is a function. Explain your answer.

Input	Outputs	Function?
(a) States	Capitals	
(b) States	Cities	
(c) Families	Pets	
(d) Families	Last names	

2. In each of the following examples, use an input-output chart to decide if the following relation is a function.

(a) Consider the following relation: multiply the input by five and then subtract seven to get the output.

Input x	Calculation	Output y
-3		
0		
6		

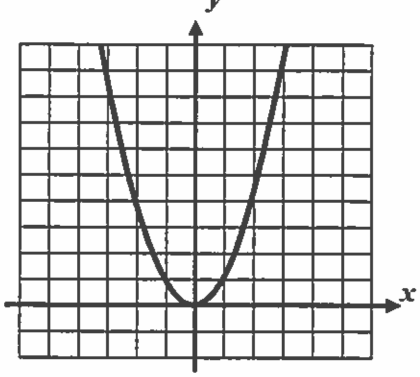
Function? Yes/No

(b) Consider the following table;

Input x	Calculation	Output y
-2	None	4
3	None	3
3	None	2

Function? Yes/No

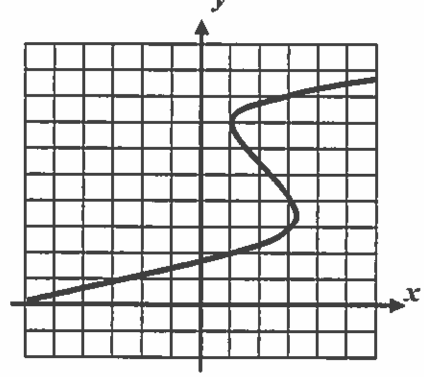
(c) Consider the following graph



Input x	Calculation	Output(s) y
-2	None	
1	None	
2	None	

Function? Yes/No

(d) Consider the following graph



Input x	Calculation	Output(s) y
-3	None	
1	None	
3	None	

Function? Yes/No

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APPLICATIONS

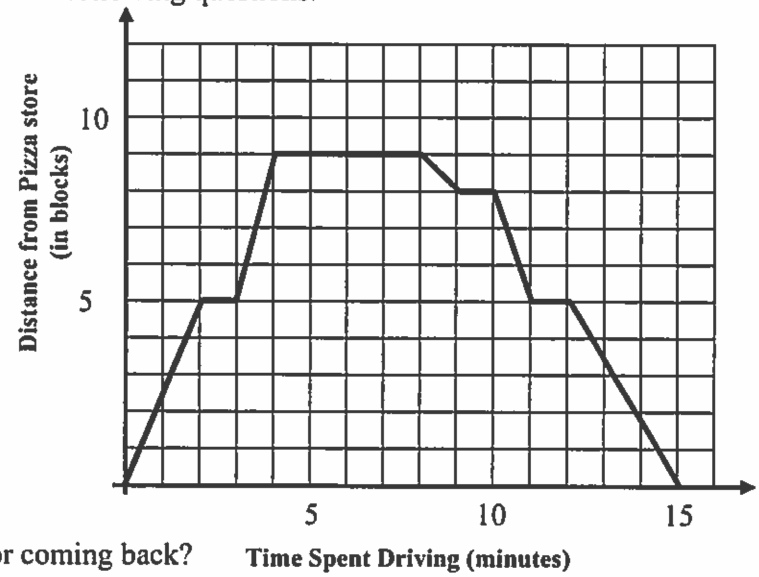
3. Andrew has a new job at the local pizza store as a delivery boy. The following graph shows one of his deliveries he made. Analyze the graph and answer the following questions.

(a) How long was the entire trip?

(b) If he arrived at the house after 4 minutes, how far away was the house from the pizza place?

(c) Why might Andrew have stopped 3 times for 1 minute?

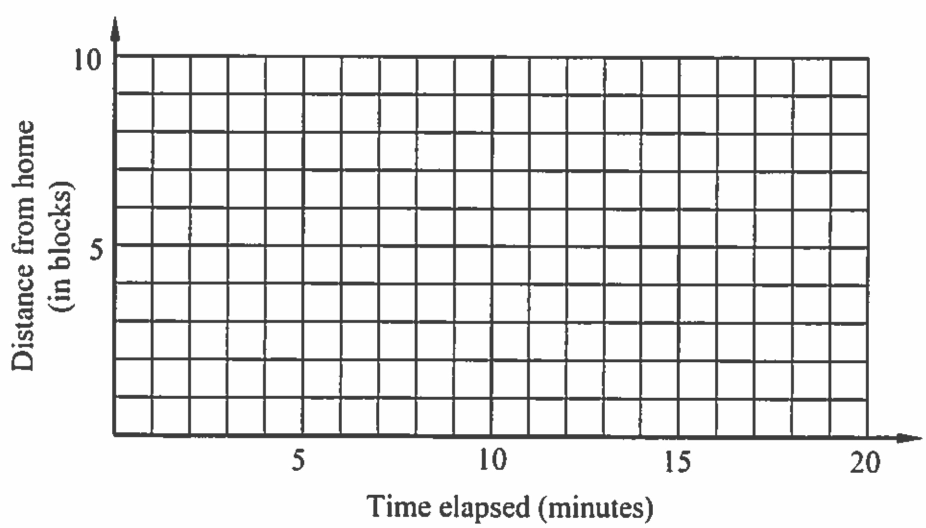
(d) Was Andrew's trip longer going to the house or coming back?



REASONING

4. Given the following scenario, graph a function that would map Liza's distance away from her house according to the time elapsed.

Liza has a few items she needs to pick up from a grocery store 8 blocks away. Liza travels as a constant rate of 2 blocks per minute when not stopped at a light. On her way to the grocery store she doesn't hit any red lights and the trip takes her 4 minutes. She spends 8 minutes in the grocery store and then starts to head home. When she's halfway home she hits a red light that lasts 3 minutes. After the light ends, she then drives the second half of the way home.



Name: ______________________________ Date: ________________

FUNCTION NOTATION

Since functions are rules that convert **inputs** (typically x-values) into **outputs** (typically y-values), it makes sense that they must have their own **notation** to indicate what the what the rule is, what the input is, and what the output is. In the first exercise, your teacher will explain how to interpret this notation.

***Exercise* #1:** For each of the following functions, find the outputs for the given inputs.

(a) $f(x)=3x+7$

$f(2)=$

$f(-3)=$

(b) $g(x)=\dfrac{x-6}{2}$

$g(20)=$

$g(0)=$

(c) $h(x)=\sqrt{2x+1}$

$h(4)=$

$h(0)=$

Function notation can be very, very confusing because it really looks like multiplication due to the parentheses. But, there is no multiplication involved. The notation serves two purposes: (1) to tell us what the rule is and (2) to specify an output for a given input.

FUNCTION NOTATION

$$y = f\ (x)$$

Output → y Rule f Input → x

***Exercise* #2:** Given the function $f(x)=\dfrac{x}{3}+7$ do the following.

(a) Explain what the function rule does to convert the input into an output.

(b) Evaluate $f(6)$ and $f(-9)$.

(c) Find the input for which $f(x)=13$. Show the work that leads to your answer.

(d) If $g(x)=2f(x)-1$ then what is $g(6)$? Show the work that leads to your answer.

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Recall that function rules commonly come in one of three forms: (1) equations (as in Exercise #1), (2) graphs, and (3) tables. The next few exercises will illustrate function notation with these three forms.

***Exercise* #3:** Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, T, is a function of the number of hours, h.

h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ $(^\circ F)$	212	141	104	85	76	70	68	66	65

(a) Evaluate $T(2)$ and $T(6)$.

(b) For what value of h is $T(h)=76$?

(c) Between what two consecutive hours will $T(h)=100$? Explain how you arrived at your answer.

***Exercise* #3:** The function $y=f(x)$ is defined by the graph shown below. It is known as **piecewise linear** because it is made up of **straight line segments**. Answer the following questions based on this graph.

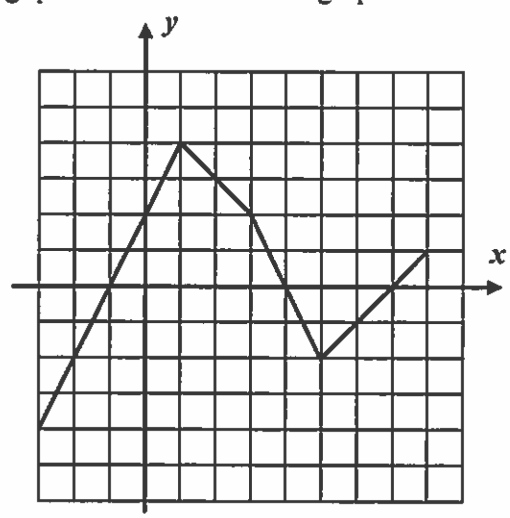
(a) Evaluate each of the following:

$f(1)=$ $\qquad$ $f(5)=$

$f(-3)=$ $\qquad$ $f(0)=$

(b) Solve each of the following for all values of the input, x, that make them true.

$f(x)=0$ $\qquad$ $f(x)=2$



(c) What is the largest output achieved by the function? At what x-value is it hit?

Name: ______________________________ Date: ________________

FUNCTION NOTATION

FLUENCY

1. Given the function *f* defined by the formula $f(x) = 2x + 1$ find the following:

   (a) $f(4)$ (b) $f(-5)$ (c) $f(0)$ (d) $f\left(\frac{1}{2}\right)$

2. Given the function *g* defined by the formula $g(x) = \frac{x-5}{2}$ find the following:

   (a) $g(9)$ (b) $g(0)$ (c) $g(3)$ (d) $g(17)$

3. Given the function *f* defined by the formula $f(x) = x^2 - 4$ find the following:

   (a) $f(3)$ (b) $f(-4)$ (c) $f(0)$ (d) $f(-2)$

4. If the function $f(x)$ is defined by $f(x) = \frac{x}{2} - 6$ then which of the following is the value of $f(10)$?

   (1) -1 (3) 14

   (2) 2 (4) 7

5. If the function $f(x) = 2x - 3$ and $g(x) = \frac{3}{2}x + 1$ then which of the following is a true statement?

   (1) $f(0) > g(0)$ (3) $f(8) = g(8)$

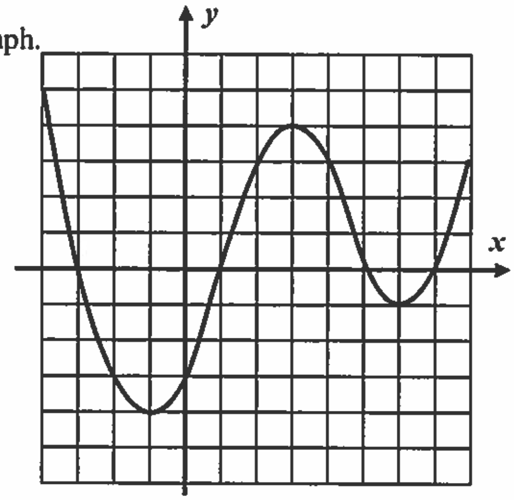
   (2) $f(2) = g(2)$ (4) $g(4) < f(4)$

6. Based on the graph of the function $y = g(x)$ shown below, answer the following questions.

(a) Evaluate each of the following. Illustrate with a point on the graph.

$g(-2) =$ $g(0) =$

$g(3) =$ $g(7) =$



(b) What values of x solve the equation $g(x) = 0$? These are called the **zeroes of the function**

(c) How many values of x solve the equation $g(x) = 2$? How can you illustrate your answer on the graph? Remember, we are not looking for the exact x-values, only **how many solutions**.

APPLICATIONS

6. Physics students drop a ball from the top of a 100 foot high building and model its height above the ground as a function of time with the equation $h(t) = 100 - 16t^2$. The height, h, is measures in feet and time, t, is measured in seconds. Be careful with all calculations in this problems and remember to do the exponent (squaring) first.

(a) Find the value of $h(0)$. Include proper units. What does this output represent? Reread the problem if necessary.

(b) Calculate $h(2)$. Does our equation predict that the ball has hit the ground at 2 seconds? Explain.

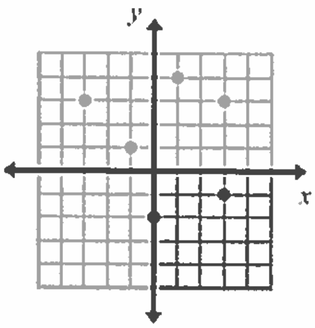
REASONING

7. If you knew that $f(-4) = 8$, then what (x, y) coordinate point must lie on the graph of $y = f(x)$? Explain your thinking.

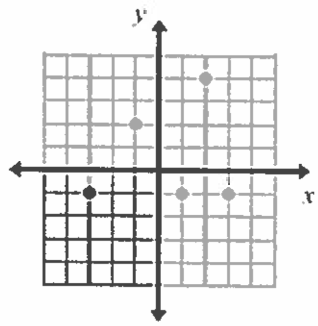
Name: ______________________________ Date: ____________________

Decide whether the graph is a function or relation. If it is a function, give the domain and range.

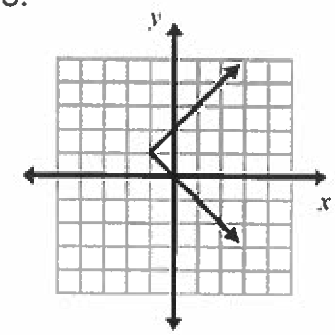
1.



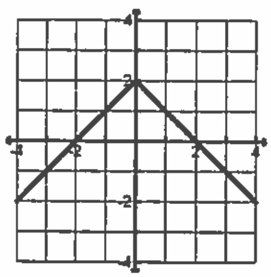
2.



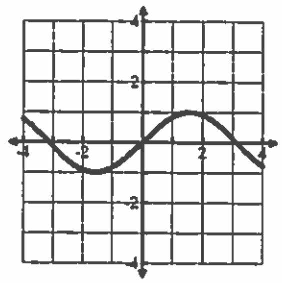
3.



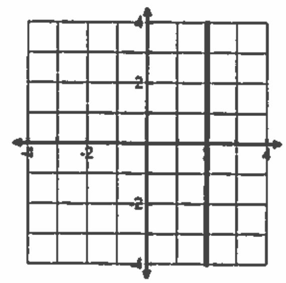
4.



5.



6.



Decide whether the relation is a function.
If it is a function, give the domain and the range.

7.

Input	Output
1 →	7
1 →	-7
2 →	8
2 →	-8

8.

Input	Output
3 →	2
5 →	4
7 →	4
	6

9.

Input	Output
0 →	-6
2 →	-4
4 →	-2
6 →	0

Evaluate the function when $x = 3$, $x = 0$, and $x = -2$. (3 answers for each problem)

10. $f(x) = 2x - 5$

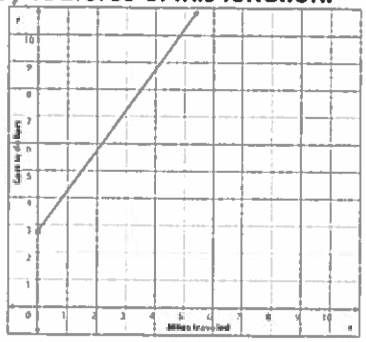
11. $h(x) = 6x + 2$

12. $g(x) = 2.4x$

Name: ____ Date: ____________________

Characteristics of Functions Practice

1. A taxi company in Atlanta charges $2.75 per ride plus $1.50 for every mile driven. Write the equation for the line, and determine the key features of this function.



Equation: __________ Discrete or Continuous: __________

Domain: __________ Range: __________

Intercepts: __________ Increasing or Decreasing: __________

Max or Min: __________

2. A pendulum swings to 90% of its height on each swing and starts at a height of 80 cm. The height of the pendulum in centimeters, y, is recorded after x number of swings. Write the equation, and determine the key features of this function.

Number of swings (x)	Height in cm (y)
0	80
1	72
2	64.8
3	58.32
5	47.24
10	27.89
20	9.73
40	1.18
60	0.14
80	0.02

Equation: __________ Discrete or Continuous: __________

Domain: __________ Range: __________

Intercepts: __________ Increasing or Decreasing: __________

Max or Min: __________

3. The cost of an air conditioner is $110. The cost to run the air conditioner is $0.35 per minute. Write the equation, and determine the key features of this function.

Minutes (x)	Cost in dollars ($f(x)$)
0	110.00
30	120.50
60	131.00
90	141.50
120	152.00

Equation: __________ Discrete or Continuous: __________

Domain: __________ Range: __________

Intercepts: __________ Increasing or Decreasing: __________

Max or Min: __________

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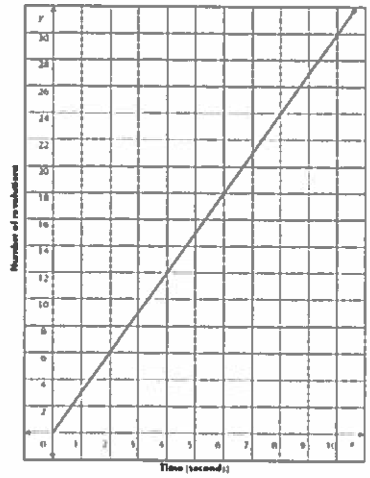
4. **A gear on a machine turns at a rate of 3 revolutions per second. Write the equation, and determine the key features of this function.**



Equation: ____________

Discrete or Continuous: ____________

Domain: ____________

Range: ____________

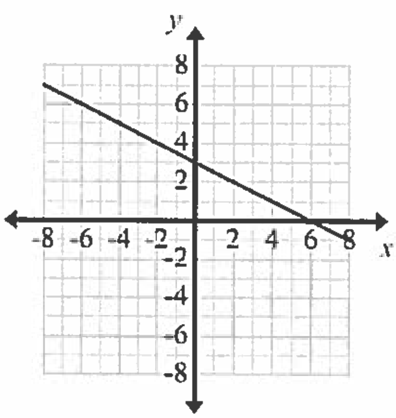
Intercepts: ____________

Increasing or Decreasing: ________

Max or Min: ____________

5. **Fill in the information for each graph.**

a)



Domain: ____________

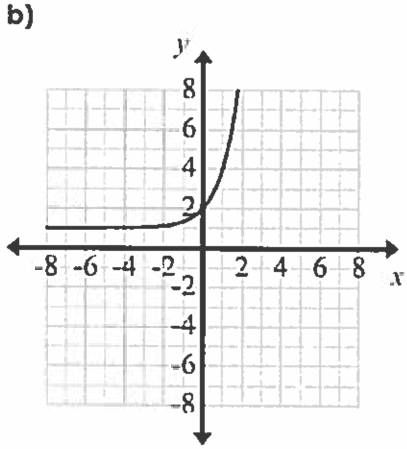
Range: ____________

Intercepts: ____________

Increasing / Decreasing: ______

Max or Min: ____________

b)



Domain: ____________

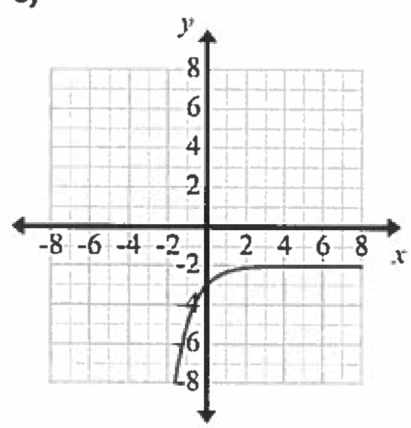
Range: ____________

Intercepts: ____________

Increasing / Decreasing: ______

Max or Min: ____________

c)



Domain: ____________

Range: ____________

Intercepts: ____________

Increasing / Decreasing: ____

Max or Min: ____________

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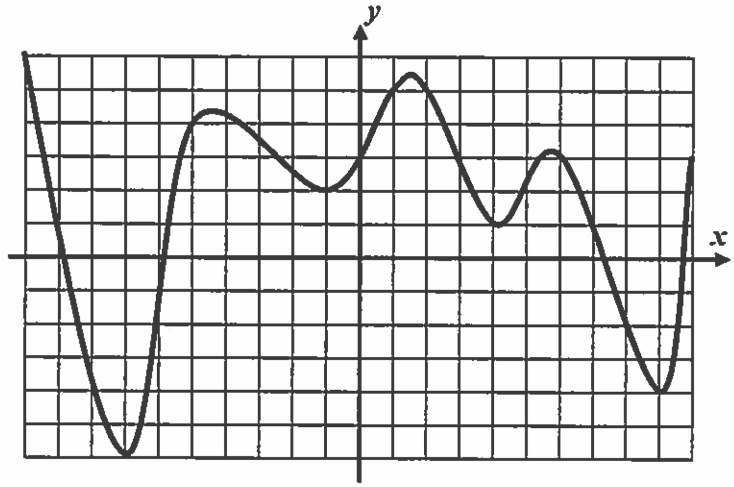
Name: ______________________________ Date: ________________

GRAPHS OF FUNCTIONS

FLUENCY

1. Using the graph of the function $f(x)$ shown below, answer the following questions.



(a) Find the value of each of the following:

$f(-7)=$ $f(0)=$

$f(4)=$ $f(9)=$

(b) For how many values of x does $f(x)=5$? Illustrate on the graph.

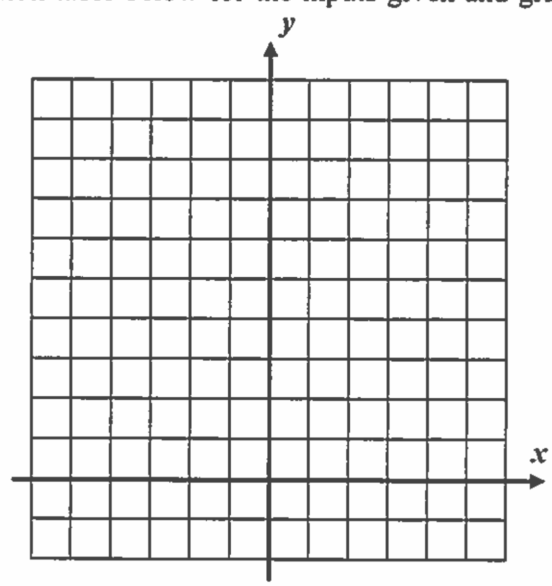
(c) What is the y-intercept of this relation?

(d) State the maximum and minimum values the graph obtains.

(e) Explain why the graph above represents a function.

2. Consider the function $f(x)=3(2-x)-2$. Fill out the function table below for the inputs given and graph the function on the axes provided.

x	$3(2-x)-2$	(x, y)
-2		
-1		
0		
1		
2		



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APPLICATIONS

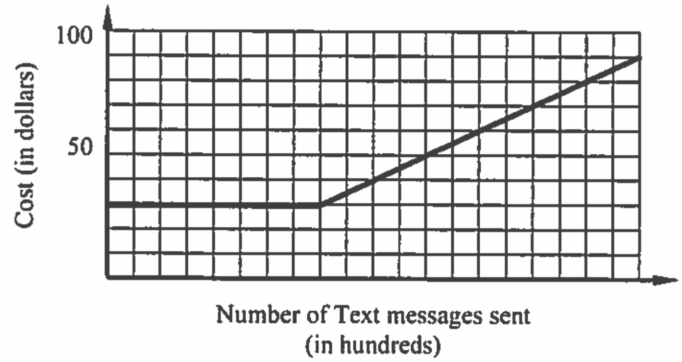
3. The following graph represents the cost of a phone plan after a certain number of text messages used in a month. Analyze the graph to answer the following questions.

(a) How much would you have to pay if you used:

500 text messages ___________

1800 text messages ___________

(b) Interpret $f(1400) = 60$



(c) What might have caused the graph to begin increasing at 800 text messages?

REASONING

4. Consider the following relationship given by the formula $f(x) = \begin{cases} 3-2x & x \le 1 \\ 2x-1 & x > 1 \end{cases}$.

(a) Evaluate each of the following:

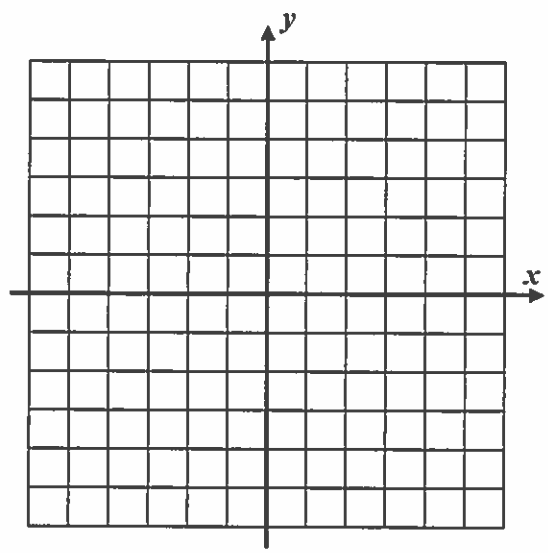
$f(5) =$ $\qquad\qquad$ $f(-2) =$

(b) Carefully evaluate $f(1)$.

(c) Fill out the table below for the inputs given. Keep in mind which formula you are using.

x	Rule/Calculation	(x, y)
-1		
0		
1		
2		
3		

(d) Graph $y = f(x)$ on the axes below.



(e) What is the minimum value of the function? Circle the point that indicates this value on the graph.

Algebra 1 Functions

Name:________________________

Describe the attributes of a linear function.

Block:_________ Date: ____________

Part 1: Function: $f(x) = 2x + 3$

Attribute	Write your answers in complete sentences.
Rate of Change (slope)	
Domain	
Range	
x-intercept	
y-intercept	
End Behavior	

Part 2: Compare and Contrast

Give an example of a function and an example of an equation that is NOT a function.	Funtion:	Not a Function:
How are they the same?		
How are they different?		