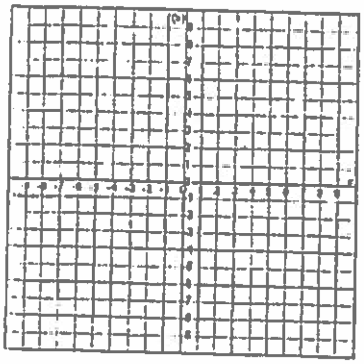
Unit 3: Systems of Equations and Inequalities

$y = mx + b$

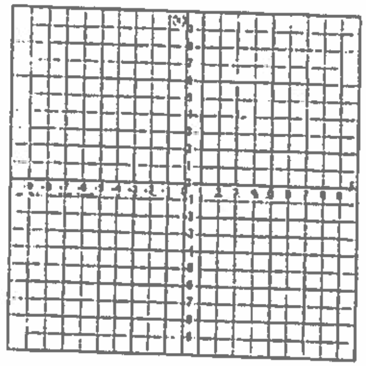
↑slope ↑y-intercept

Graph the following linear equations using slope-intercept form.

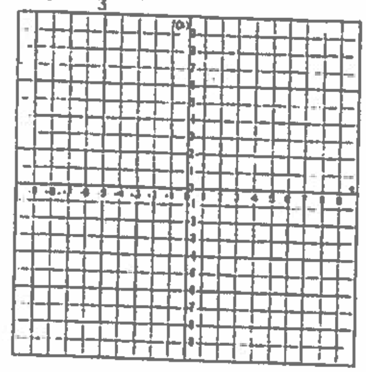
1. $y = 2x + 1$



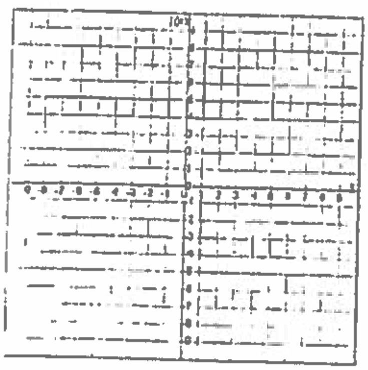
2. $y = 3x - 4$



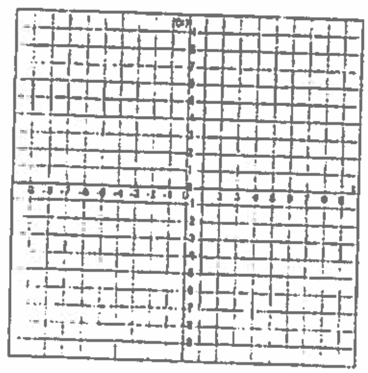
3. $y = \frac{2}{3}x + 5$



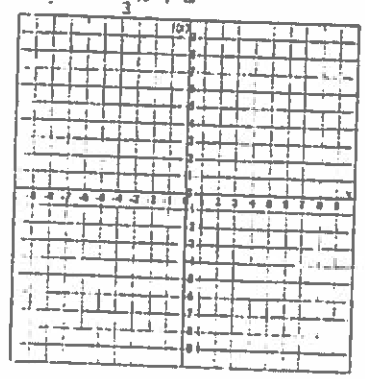
4. $y = 7$



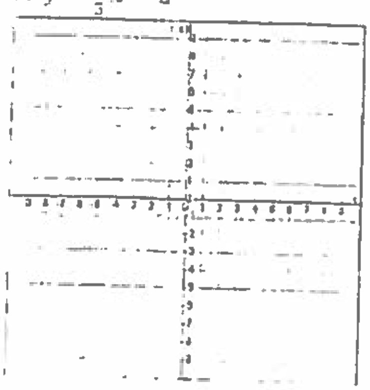
5. $y = -3x - 2$



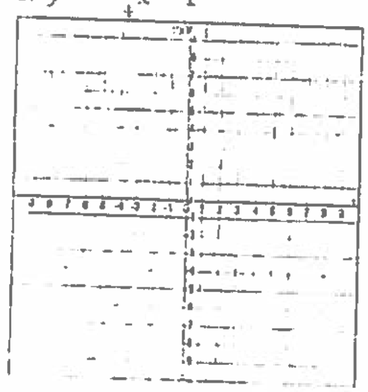
6. $y = -\frac{1}{3}x + 5$



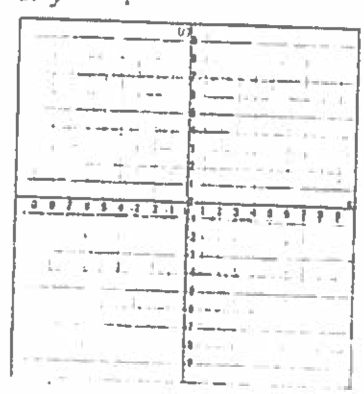
7. $y = \frac{2}{3}x - 2$



8. $y = -\frac{3}{4}x - 1$



9. $y = -4$



Graphing in
lope intercept
rm:
$=mx+b$

① identify
slope(m)
y-intercept(b)

② plot y-int
on y-axis

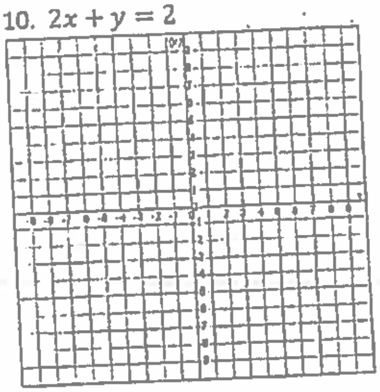
③ count slope (and plot points in
both directions)

$m = \frac{rise}{run}$

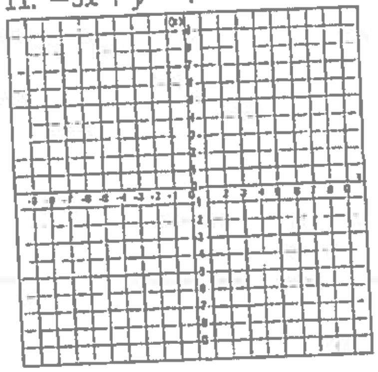
④ draw line
throug points

2

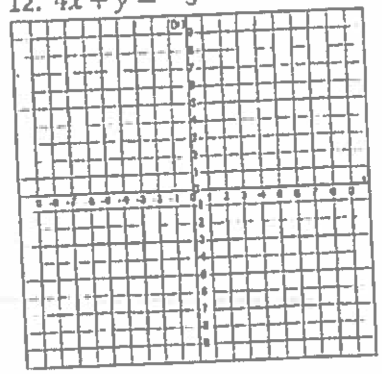
10. $2x + y = 2$



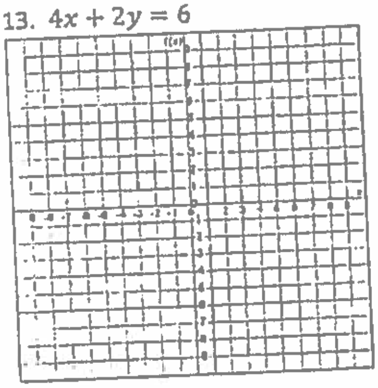
11. $-3x + y = 4$



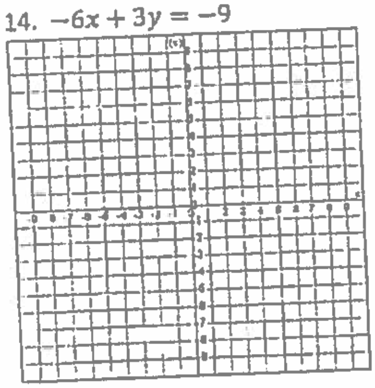
12. $4x + y = -5$



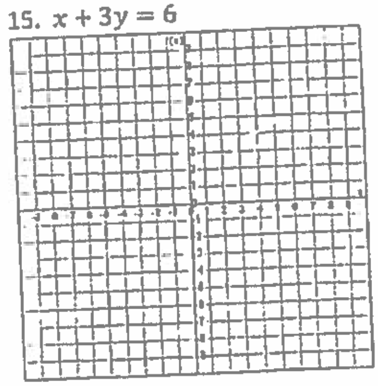
13. $4x + 2y = 6$



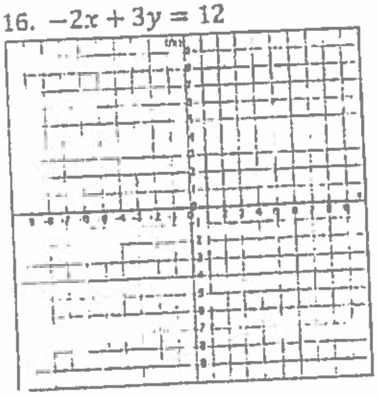
14. $-6x + 3y = -9$



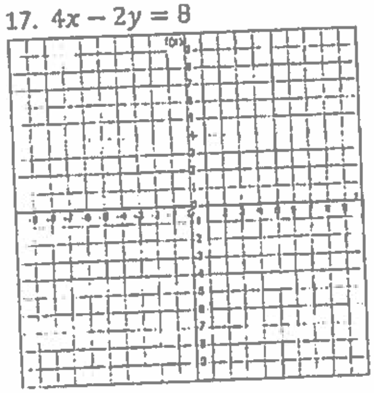
15. $x + 3y = 6$



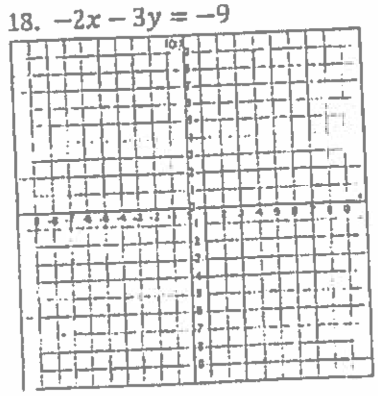
16. $-2x + 3y = 12$



17. $4x - 2y = 8$



18. $-2x - 3y = -9$



Solve equations 10-18 for y first:

(10) y= ______ (13) y= ______ (16) y= ______

(11) y= ______ (14) y= ______ (17) y= ______

(12) y= ______ (15) y= ______ (18) y= ______

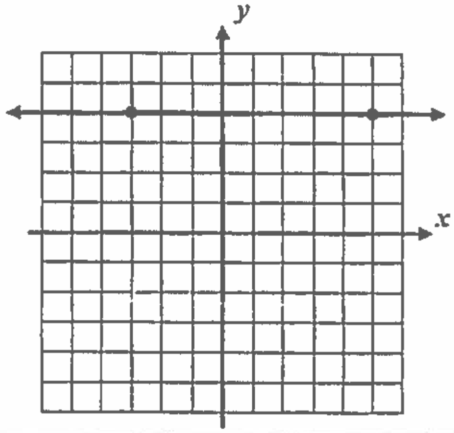
4

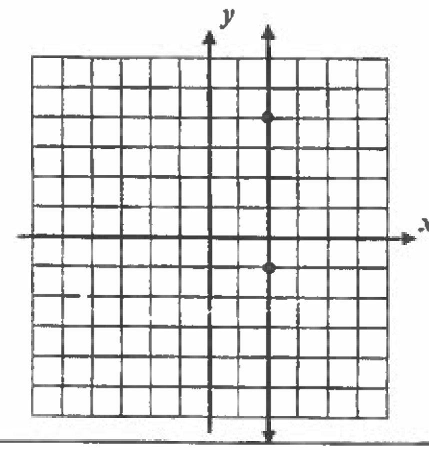
Name: ______________________ Date: ______________

STRANGE LINES – VERTICAL AND HORIZONTAL

Although they don't fit the classic linear model, it is important to understand how we write equations for **horizontal and vertical lines.** The first exercise will illustrate the idea. Never forget, though, that when we create an equation for a curve, it simply **describes what all points on the curve share in common.**

Exercise #1: Shown below are a horizontal line and a vertical line.





HORIZONTAL LINE

Write down two coordinate points:

What do they share in common?

What is this line's equation?

VERTICAL LINE

Write down two coordinate points:

What do they share in common?

What is this line's equation?

Equations of horizontal lines and vertical lines are so simple that students will often get them confused later, because they don't really seem like typical linear equations (because they aren't).

HORIZONTAL AND VERTICAL LINES

Horizontal Line: $y=\text{constant}$ **Vertical Line:** $x=\text{constant}$

(Constants can be determined by using any point the line passes through)

Exercise #2: Which of the following equations represents a vertical line that passes through the point $(5,-3)$?

(1) $y=-3$ (3) $y=-3x+5$

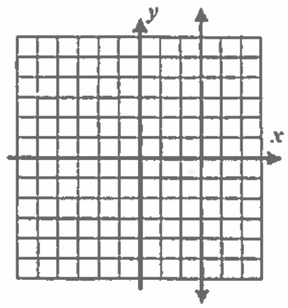
(2) $x=5$ (4) $y=5x-3$

6

It is important to be able to quickly and accurately graph vertical and horizontal lines as well as give their equations based on their graphs. We will try to build some fluency with this in the next exercise.

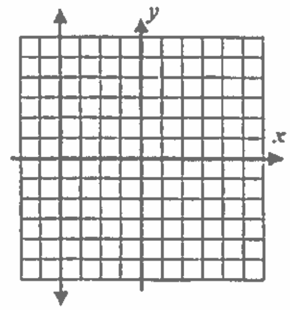
Exercise #3: For each of the following, give the equation of the line shown or described.

(a)



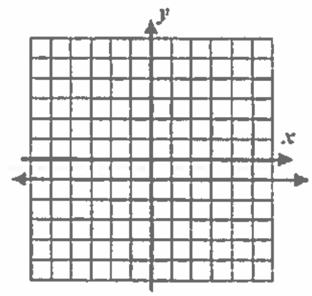
EQUATION: ____________

(b)



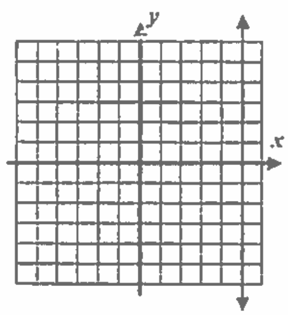
EQUATION: ____________

(c)



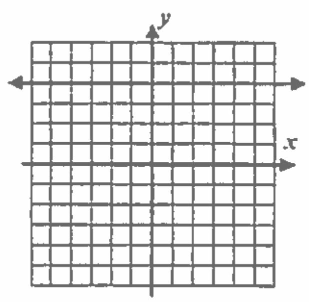
EQUATION: ____________

(d)



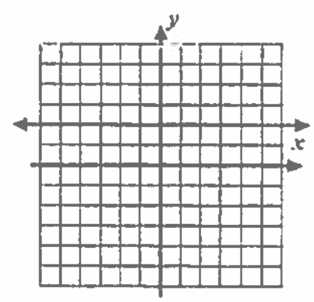
EQUATION: ____________

(e)



EQUATION: ____________

(f)



EQUATION: ____________

(g) The equation of a vertical line passing through the point $(-4, 5)$.

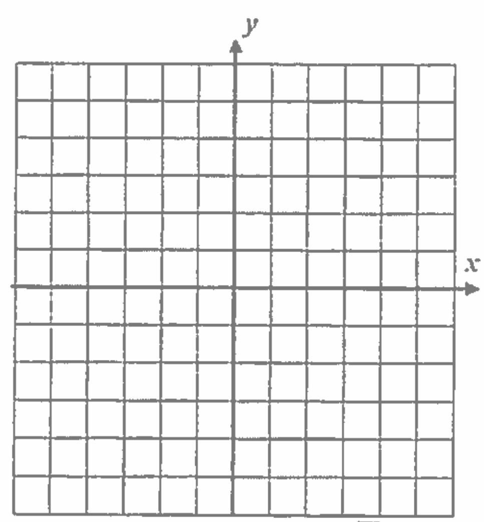
(h) The equation of a horizontal line passing through the point $(3, 2)$.

Exercise #4: Sketch the region bounded by the three lines whose equations are given below. Label each with its equation. Find the area of the triangular region enclosed by the lines. You may want to use your calculator to create a table of values of the first line or simply use facts about the slope and y-intercept.

$y = 2x - 4$

$x = -1$

$y = 2$



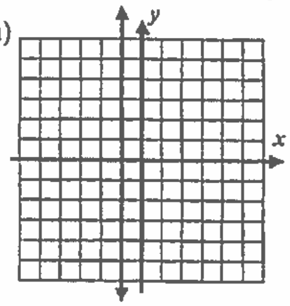
Name: ______________________________ Date: ______________

Strange Lines – Vertical and Horizontal

Fluency

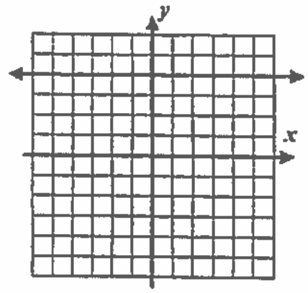
1. For each of the following, give the equation of the line shown.

(a)



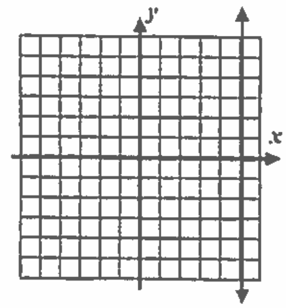
Equation: ____________

(b)



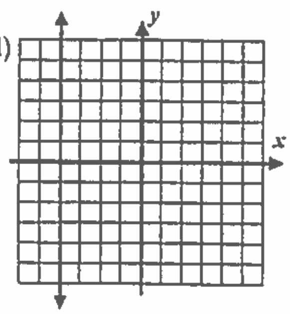
Equation: ____________

(c)



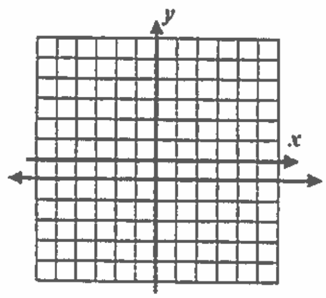
Equation: ____________

(d)



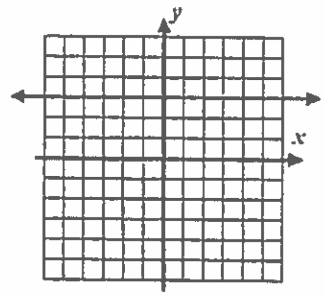
Equation: ____________

(e)



Equation: ____________

(f)



Equation: ____________

2. Write the equations of lines that fit the following descriptions. Sketch a picture if needed.

(a) A vertical line that passes through the point $(4,-7)$.

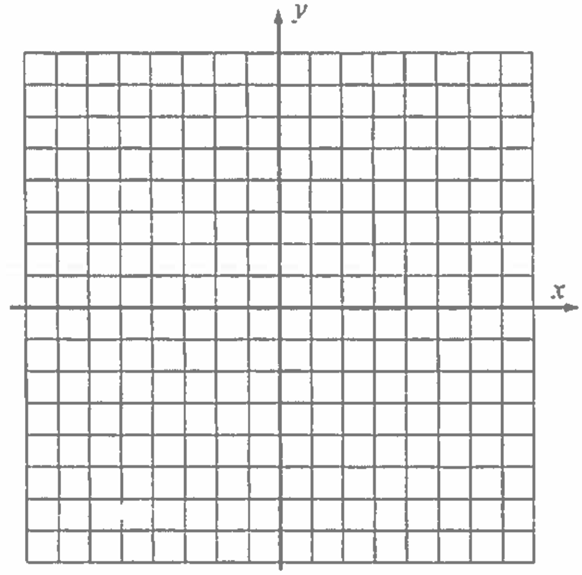
(b) A horizontal line that passes through the point $(-2,3)$.

(c) A line parallel to the x-axis that passes through the point $(-2,15)$.

(d) A line perpendicular to the x-axis that passes through the point $(5,1)$.

8

3. Each of the following lines are either horizontal, vertical, or slanted. Label each with its type and then graph on the grid. Label each with its equation.



Type:

(a) $y = \frac{3}{5}x - 2$ ________________

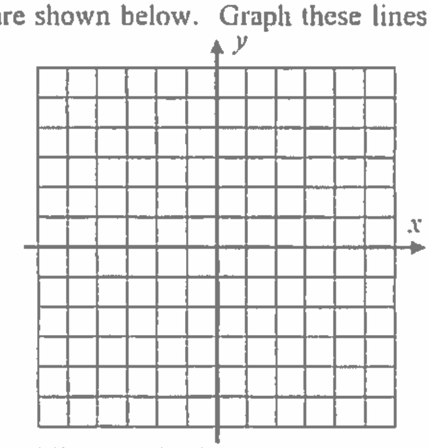
(b) $y = 6$ ________________

(c) $y = -x + 7$ ________________

(d) $x = -4$ ________________

(e) $y = 2x + 1$ ________________

4. A rectangle is surrounded by the lines whose equations are shown below. Graph these lines and find the area of the rectangle enclosed by them.



$x = -4$ $x = 3$

$y = -2$ $y = 2$

Area: ____________________

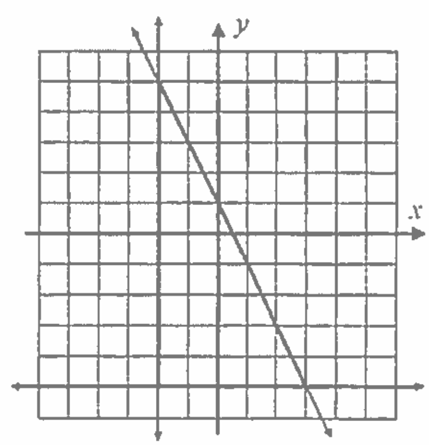
5. The triangular region shown below is bordered by one vertical line, one horizontal line, and one slanted line. State the equation of each line and determine the triangle's area.

Vertical Line: ________________

Horizontal Line: ______________

Slanted Line: __________________

Area: ____________________



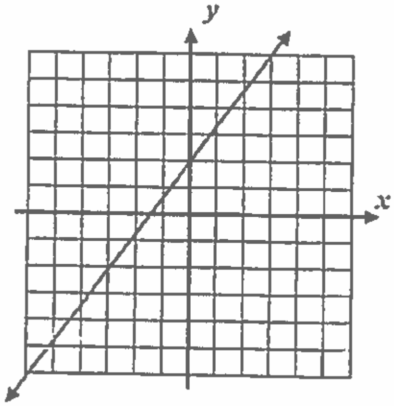
Name: ______________________ Date: ____________

WRITING EQUATIONS IN SLOPE-INTERCEPT FORM

FLUENCY

1. Each of the following lines has a slope and y-intercept that can be determined by examining the graph. For each, state the slope, the y-intercept, and then write the equation in $y = mx + b$ form (slope-intercept form).

(a)

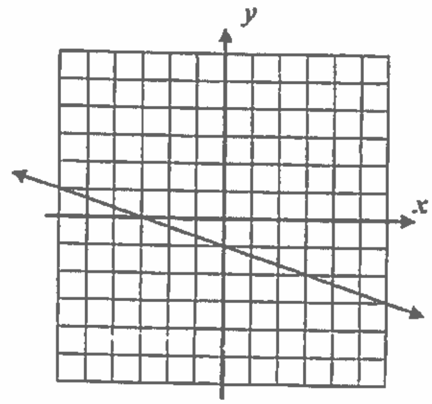


Slope: ________

y-intercept: ________

Equation: ____________

(b)



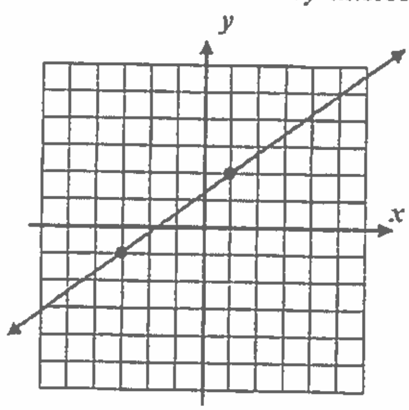
Slope: ________

y-intercept: ________

Equation: ____________

2. Each of the following lines has a slope that can be determined by examining the graph. Use another point on the line to solve for the exact y-intercept. Then, state the equation of the line.

(a)

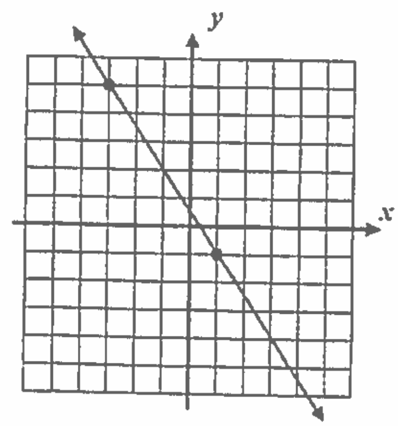


Slope: ____________

Solve for y-intercept:

Equation: ____________

(b)



Slope: ____________

Solve for the y-intercept:

Equation: ____________

3. Find the equation of the line that passes through each of the following pairs of points in $y = mx + b$ form.

(a) $(1, 7)$ and $(4, 22)$

(b) $(-2, 13)$ and $(2, 3)$

(c) $(4, 6)$ and $(10, 0)$

(d) $(0, -10)$ and $(16, 2)$

APPLICATIONS

4. A steady snow fall is coming down outside. Prestel decides to measure the depth of the snow on the ground. After 4 hours, the snow is at a depth of 9 inches an after 8 hours it is at a depth of 14 inches.

(a) Express the information given in this problem as two coordinate pairs, (h, d), where h is the number of hours and d is the depth of snow.

(b) Find the slope of the line that passes through these two points. What are its units?

(c) Find the equation of the line that passes through the two points in $d = mh + b$ form.

(d) What was the depth when the snowfall began ($h = 0$)? What would the depth be after 12 hours?

Name:_______________

Writing Equations in Context: $y = mx + b$

1. Suppose that the water level of a river is 34 feet and that it is receding at a rate of 0.5 foot per day. Write an equation for the water level, *L*, after *d* days. In how many days will the water level be 26 feet?

Equation:_______________________

2. Seth's father is thinking of buying his son a six-month movie pass for $40. With the pass, matinees cost $1.00. If matinees are normally $3.50 each, how many times must Seth attend in order for it to benefit his father to buy the pass?

Equation:___________________

3. For babysitting, Nicole charges a flat fee of $3, plus $5 per hour. Write an equation for the cost, *C*, after *h* hours of babysitting. What do you think the slope and the y-intercept represent? How much money will she make if she baby-sits 5 hours?

Equation:___________________

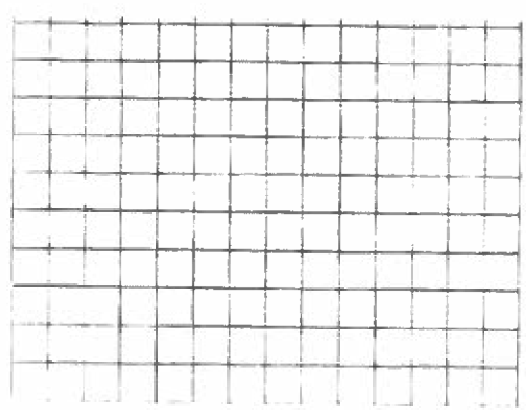
Slope represents:______________

Y-Intercept represents:_________

6. A plumber charges $25 for a service call plus $50 per hour of service. Write an equation in slope-intercept form for the cost, *C*, after *h* hours of service. What will be the total cost for 8 hours of work? 10 hours of work?

Equation:_________________

7. Rufus collected 100 pounds of aluminum cans to recycle. He plans to collect an additional 25 pounds each week. Write and graph the equation for the total pounds, *P*, of aluminum cans after *w* weeks. What does the slope and y-intercept represent? How long will it take Rufus to collect 400 pounds of cans?



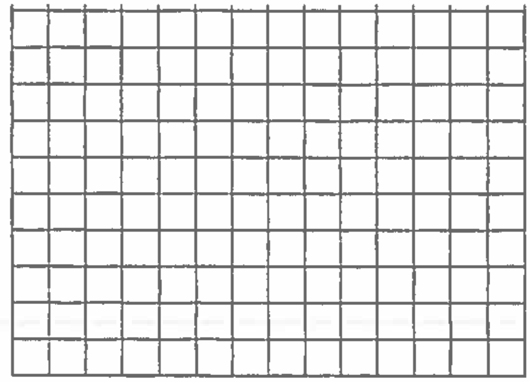
Equation:___________________

Slope represents:______________

Y-Intercept represents:_________

12

8. A canoe rental service charges a \$20 transportation fee and \$30 dollars an hour to rent a canoe. Write and graph an equation representing the cost, y, of renting a canoe for x hours. What is the cost of renting the canoe for 6 hours?



Equation:____________________

9. A caterer charges \$120 to cater a party for 15 people and \$200 for 25 people. Assume that the cost, y, is a linear function of the number of x people. Write an equation in slope-intercept form for this function. What does the slope represent? How much would a party for 40 people cost?

Equation:____________________

Slope Represents:_____________

10. An attorney charges a fixed fee on \$250 for an initial meeting and \$150 per hour for all hours worked after that. Write an equation in slope-intercept form. Find the charge for 26 hours of work.

Equation:____________________

11. A water tank already contains 55 gallons of water when Baxter begins to fill it. Water flows into the tank at a rate of 8 gallons per minute. Write a linear equation to model this situation. Find the volume of water in the tank 25 minutes after Baxter begins filling the tank.

Equation:____________________

12. A video rental store charges a \$20 membership fee and \$2.50 for each video rented. Write and graph a linear equation ($y=mx+b$) to model this situation. If 15 videos are rented, what is the revenue? If a new member paid the store \$67.50 in the last 3 months, how many videos were rented?

Equation:____________________

Name: ______________________ Date: ____________

Solutions to Linear Systems and Solving by Graphing

Systems of equations (and inequalities) are essential to modeling situations with **multiple variables and multiple relationships between the variables.** At the end of the day, though, the solution set of a system of equations can be easily defined:

Solutions to a System of Equation

1. A point (x, y) is a **solution** to a system if it makes **all equations true.**
2. The **solution set** of a system is the collection of **all** pairs (x, y) that are solutions to the system (see 1).

***Exercise* #1:** Determine if the point $(2, 5)$ is a solution to each of the systems provided. Show the work that leads to your answer for each.

(a) $y = 4x - 3$

$2x + y = 9$

(b) $y - x = 3$

$y = \frac{1}{2}x + 6$

We can solve a system by using a graph. Review this process in the next exercise.

***Exercise* #2:** Consider the system of equations shown below:

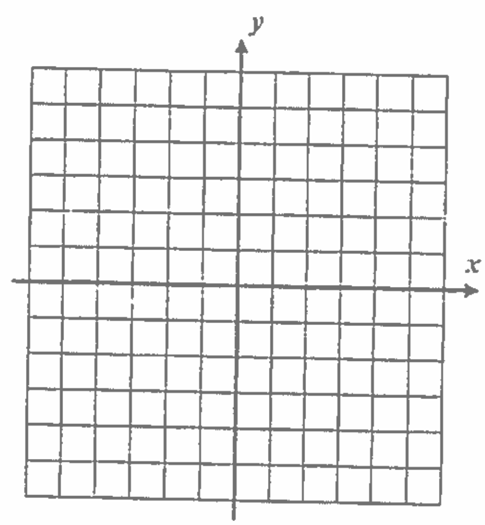
$$y = 2x + 5$$

$$y = 2 - x$$

(a) Graph both equations on the grid shown. Use TABLES on your calculator to make the process faster, if necessary. Label each line with its equation.

(b) At what point do the two lines intersect?

(c) Show that this point is a solution to the system.



14

Graphing Systems of Equations

Vocabulary:

A system of linear equations is ____

____

A solution of a system of linear equations is ____

____

Point of Intersections (POI) is the same thing as the solution of a system.

No solution means ____

____

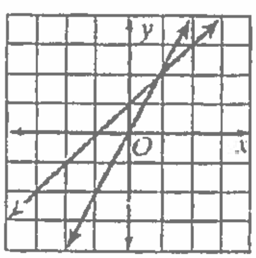
A system of equations has infinitely many solutions when ____

____

Vocabulary and Key Concepts

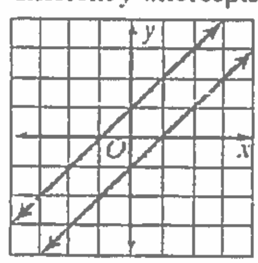
Numbers of Solutions of Systems of Linear Equations

different slopes



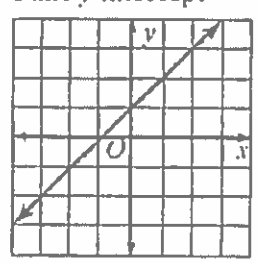
The lines ☐ so there is ☐ solution.

same slope
different y-intercepts



The lines ☐ so there are ☐ solutions.

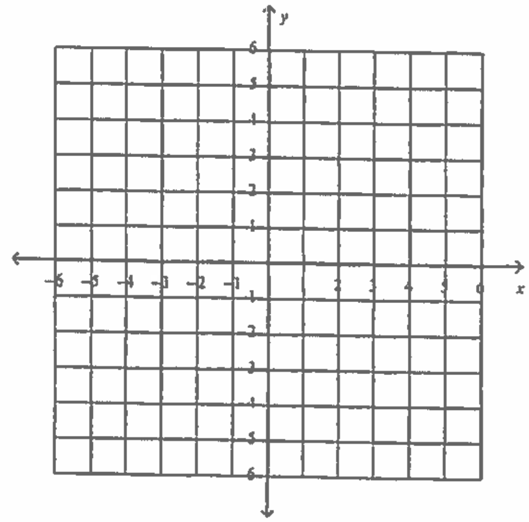
same slope
same y-intercept



The lines are ☐ so there are ☐ solutions.

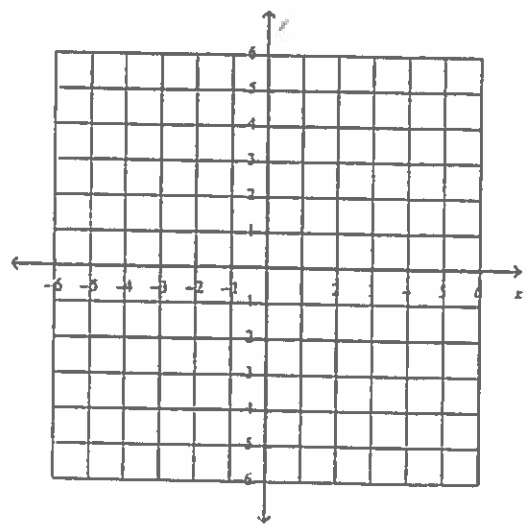
Systems with No solutions

1.) Solve by graphing: $\begin{cases} y = 3x + 2 \\ y = 3x - 2 \end{cases}$



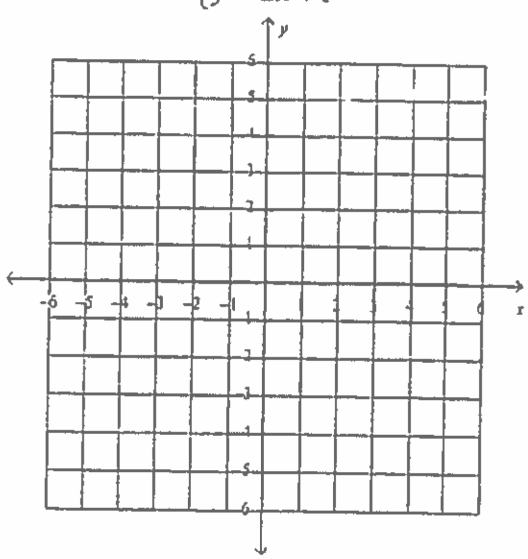
Systems with Infinitely Many solutions

2.) $\begin{cases} y = -\dfrac{3}{4}x + 3 \\ y = -\dfrac{3}{4}x + 3 \end{cases}$

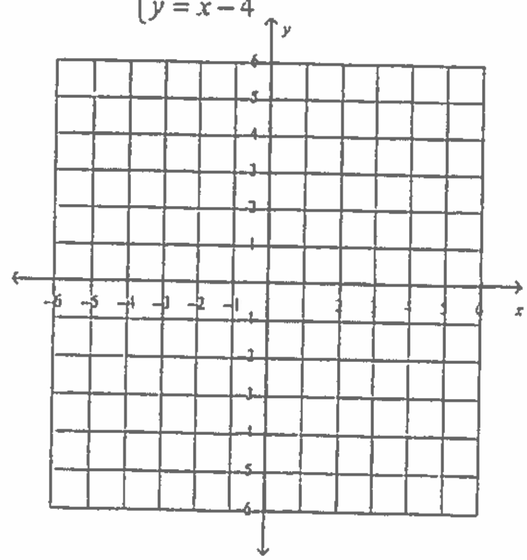


Examples:

1a.) $\begin{cases} y = x + 2 \\ y = 2x + 1 \end{cases}$

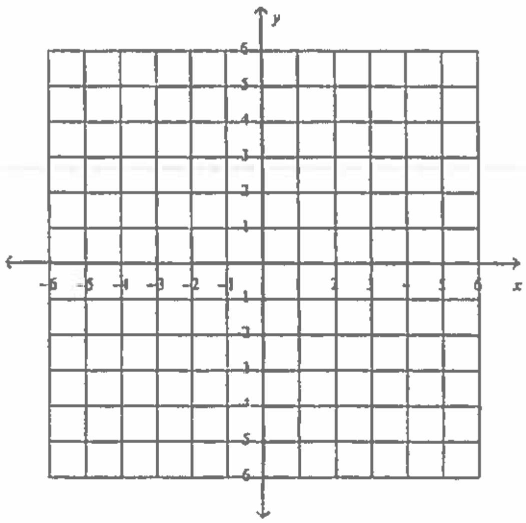


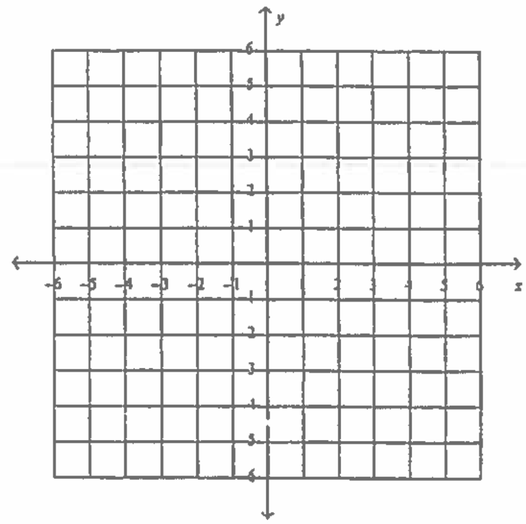
1b.) $\begin{cases} y = -\dfrac{1}{2}x - 1 \\ y = x - 4 \end{cases}$



2a.) $\begin{cases} x = 2 \\ y = -6 \end{cases}$

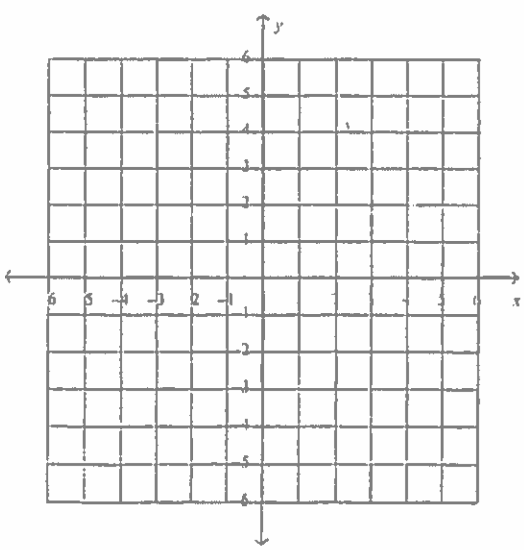
2b.) $\begin{cases} y = 3 \\ x = -4 \end{cases}$

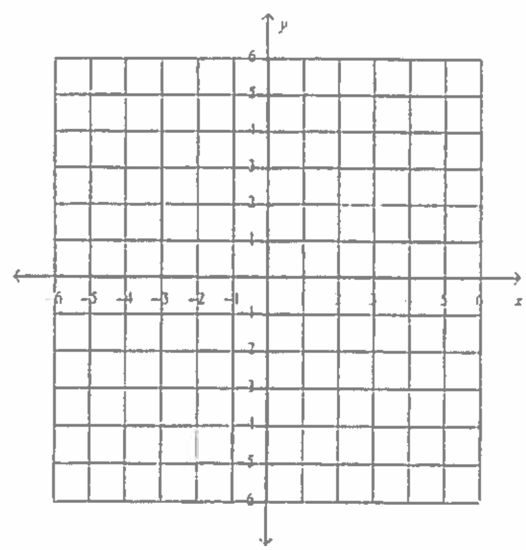




3a.) $\begin{cases} 2x - 6 = y \\ 3 - x = y \end{cases}$

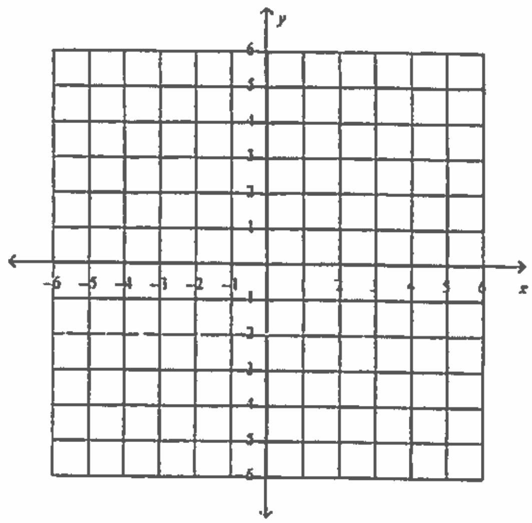
3b.) $\begin{cases} -\frac{3}{2}x + 2 = y \\ -2 + \frac{1}{2}x = y \end{cases}$



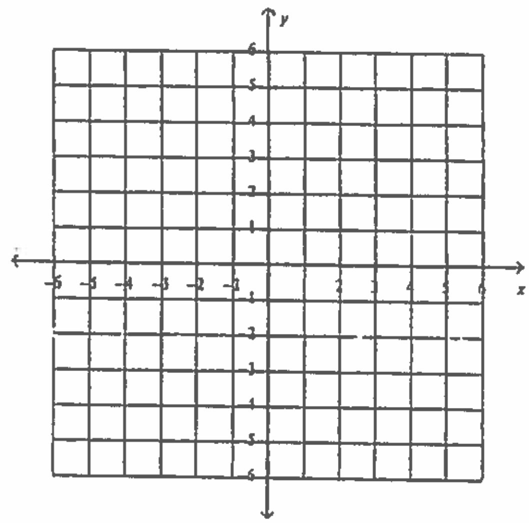


Practice:

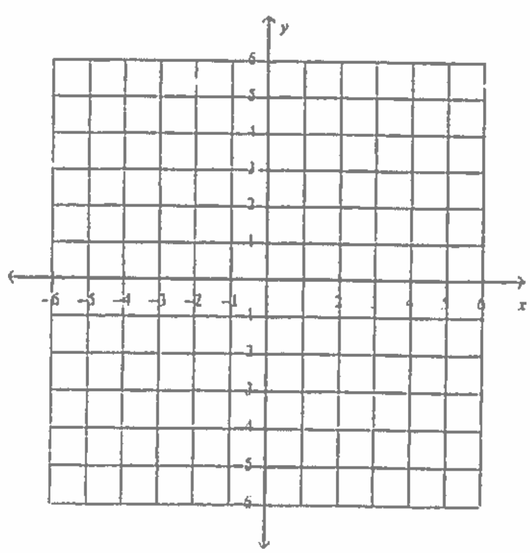
1. $\begin{cases} y = -2x + 2 \\ y = 3x + 2 \end{cases}$



2. $\begin{cases} y = 2x + 3 \\ \frac{1}{2}x = y \end{cases}$



3. $\begin{cases} y = 2x - 5 \\ y = -\frac{1}{3}x + 2 \end{cases}$



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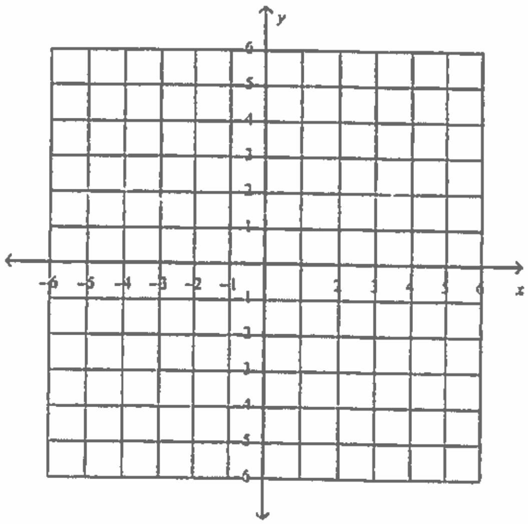
Name: ____________________ **Class:** ____________________

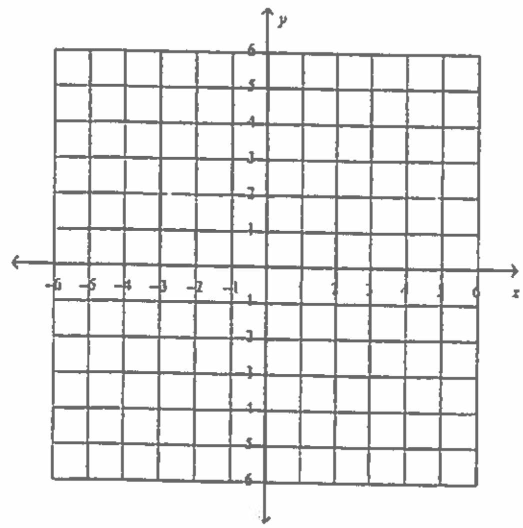
HW #1 – Graphing Systems of Equations **Date:** ____________________

What is the <u>**solution**</u> to the following system of linear equations?
If there is *no solution* or *infinitely many*, explain why.

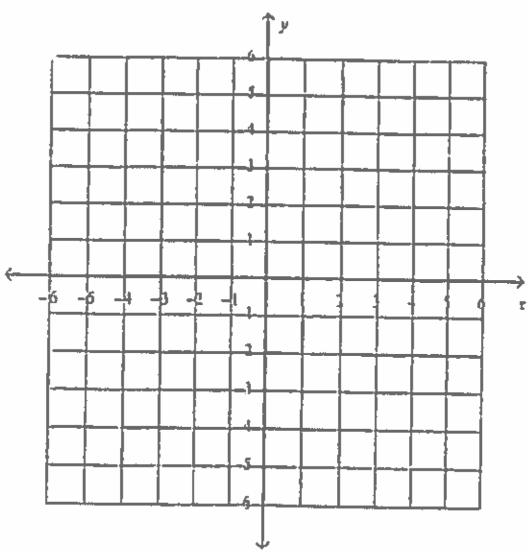
1) $\begin{cases} y = x + 3 \\ y = -2x + 3 \end{cases}$



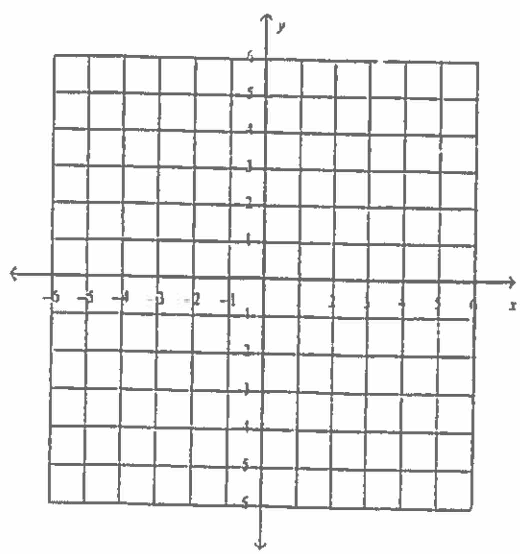
2) $\begin{cases} y = x + 2 \\ y = 4x - 1 \end{cases}$



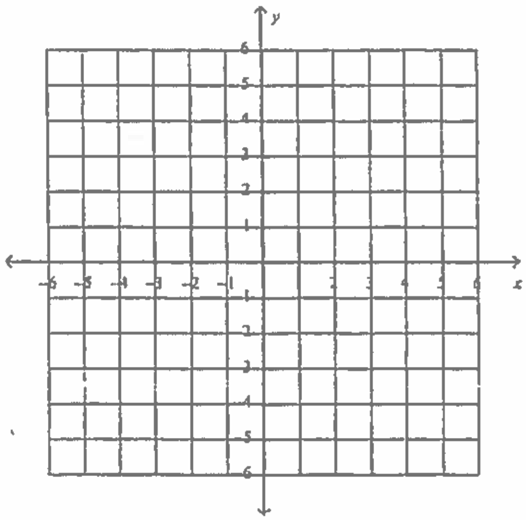
3) $\begin{cases} y = 2x + 3 \\ y = \frac{1}{2}x \end{cases}$



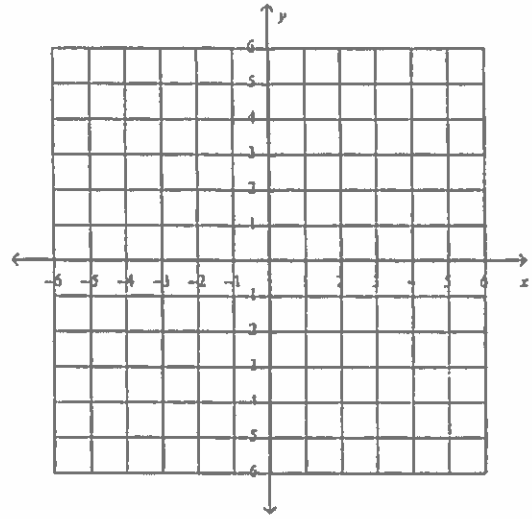
4) $\begin{cases} y = -\frac{3}{2}x + 2 \\ y = \frac{1}{2}x - 2 \end{cases}$



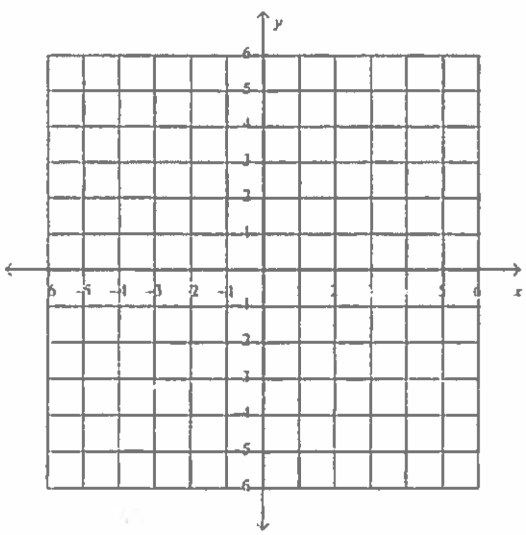
5) $\begin{cases} x = 5 \\ y = 2 \end{cases}$



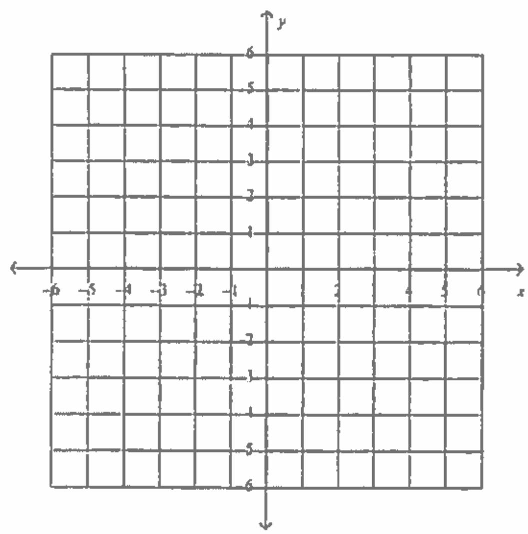
6) $\begin{cases} 2x - 5 = y \\ -1 + x = y \end{cases}$



7) $\begin{cases} y = 2x + 4 \\ y = 2x + 4 \end{cases}$



8) $\begin{cases} y = 2x - 2 \\ y = 2x + 5 \end{cases}$



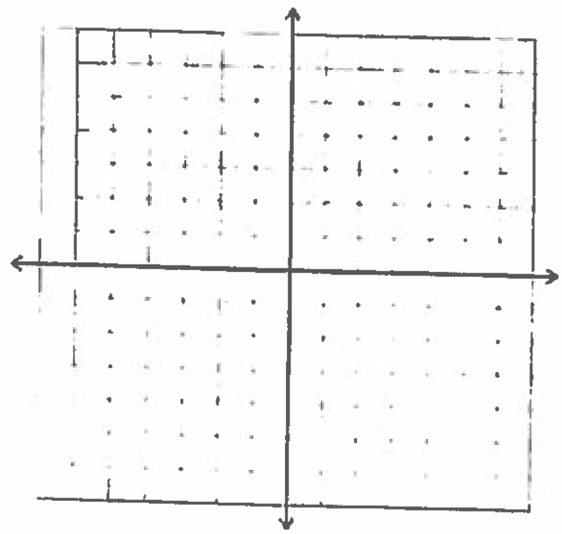
Name: ______________________ Class: ________________

Notes #2 – Graphing Systems Date: ________________

Warm-Up:

Graph the two linear equations and **find the solution.**

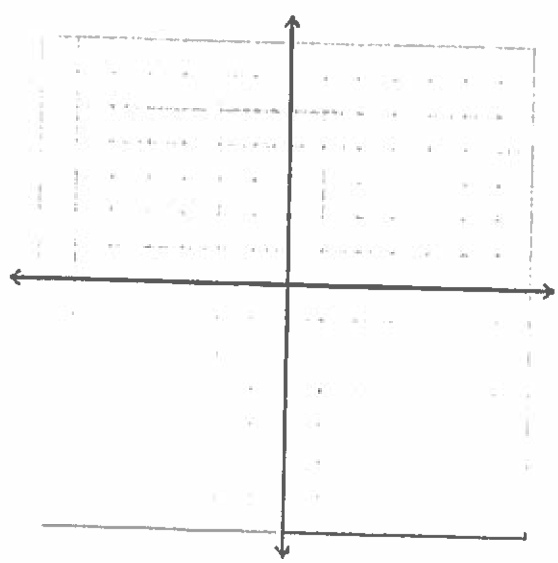
$$\begin{cases} y = -3x + 6 \\ y = -\dfrac{1}{2}x + 1 \end{cases}$$



Graphing a system of equations in different forms:

1. Find the solution to the system.

$$\begin{cases} y = 2x - 3 \\ 2x + y = 5 \end{cases}$$



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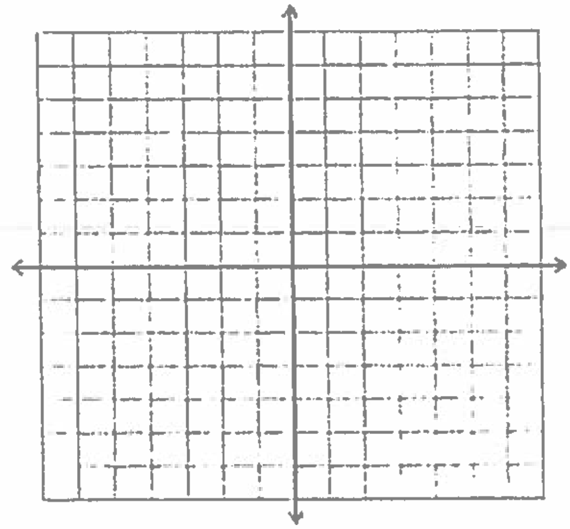
2. **Find the solution to the system.**

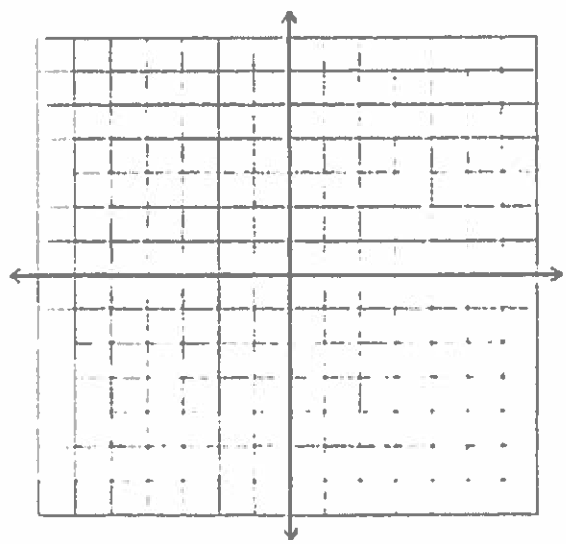
$$\begin{cases} y = -x \\ y + 3 = 2x \end{cases}$$



Try It!

Find the solution the following system.

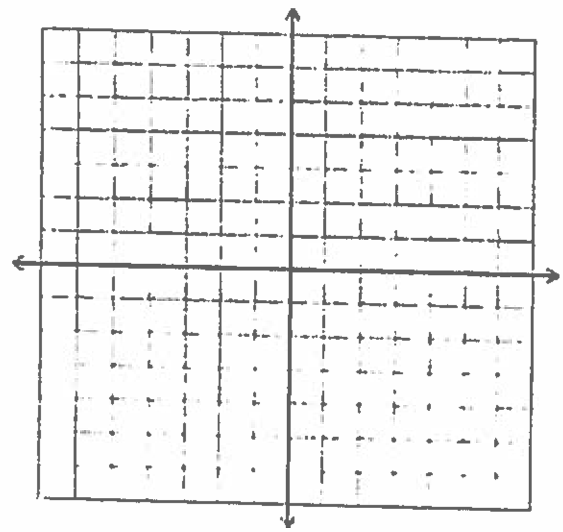
$$\begin{cases} 2x + y = -4 \\ y = 2x + 4 \end{cases}$$



Try It!

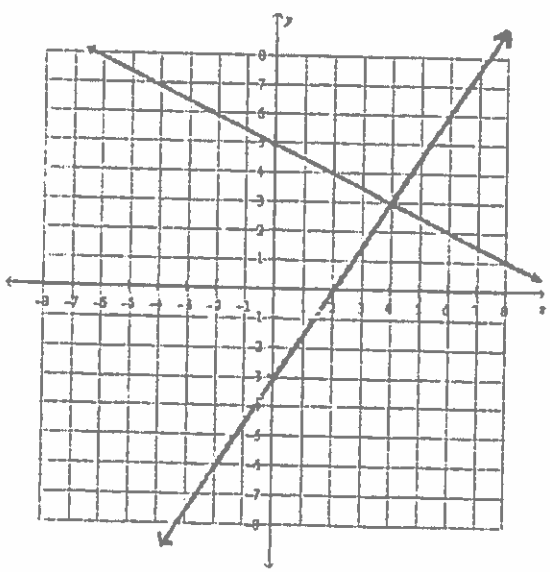
Find the solution the following system.

$$\begin{cases} -4x + y = 1 \\ y = -\dfrac{1}{2}x + 1 \end{cases}$$

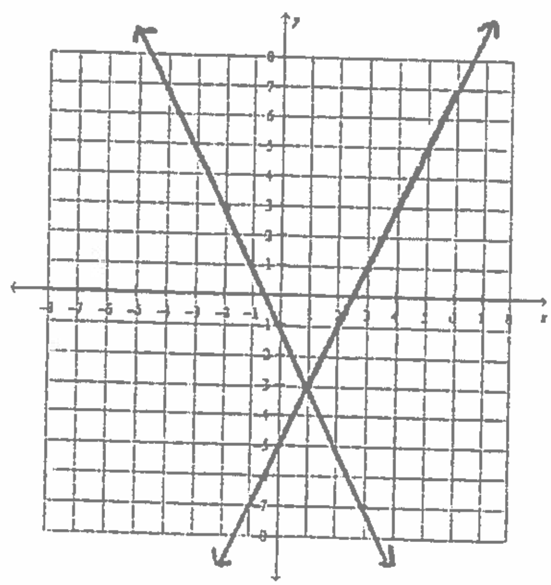


Find the solution to the given systems.

3.



4.



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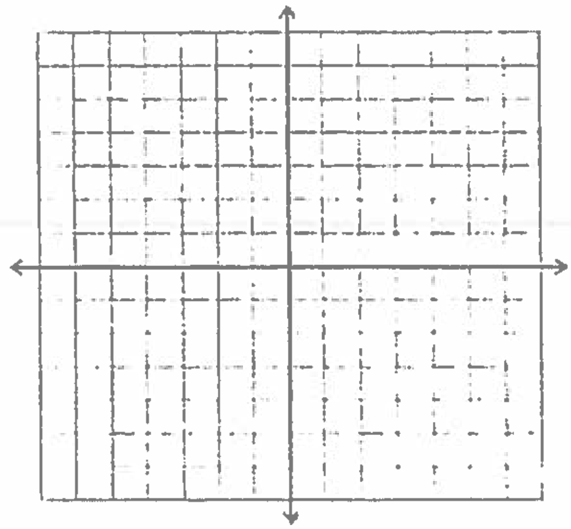
5. **Find the solution the following system.**

$$\begin{cases} 3x + 4y = 12 \\ y = -\frac{3}{4}x + 3 \end{cases}$$



Determine where the two equations intersect. Set the equations equal and solve. Make sure you find both the x and y coordinates.

6. $y = 2x + 1$
   $y = 3x - 4$

7. $2x - 1 = y$
   $4x + 2 = y$

8. $x = 3y - 4$
   $x = -3y + 2$

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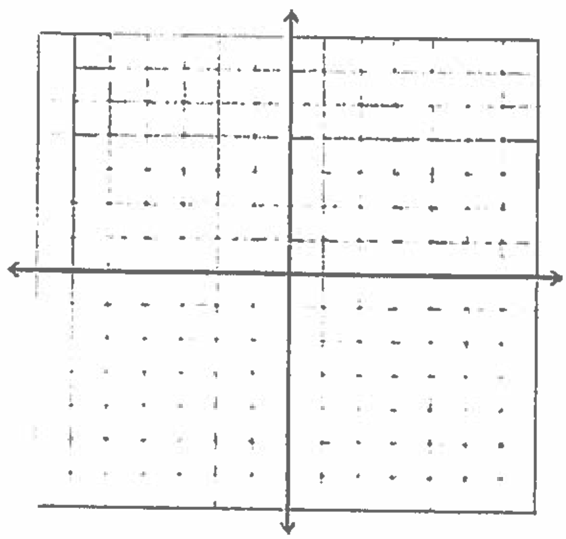
Name: ____________________ Class: ____________________

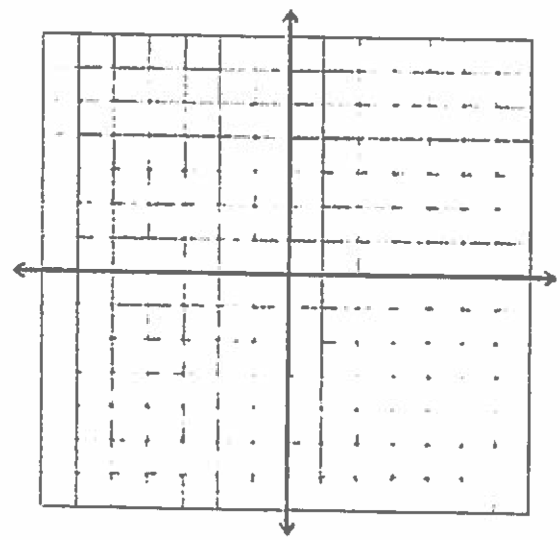
HW #2 – Graphing Systems (Day2) Date: ____________________

Graph the two linear equations and find the solution.

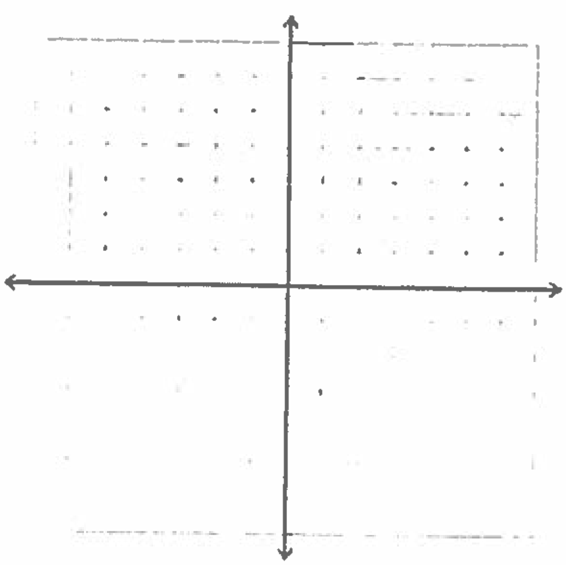
1. $\begin{cases} y = -x + 2 \\ -x + y = -2 \end{cases}$



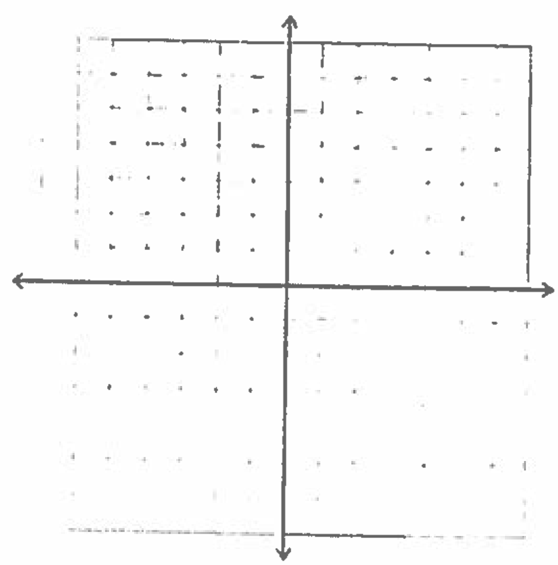
2. $\begin{cases} y = -\frac{1}{4}x - 1 \\ y - 2 = \frac{1}{2}x \end{cases}$



3. $\begin{cases} y = 2x + 6 \\ -2x + y = 6 \end{cases}$



4. $\begin{cases} y - 4 = 2x \\ y - 2x = 4 \end{cases}$



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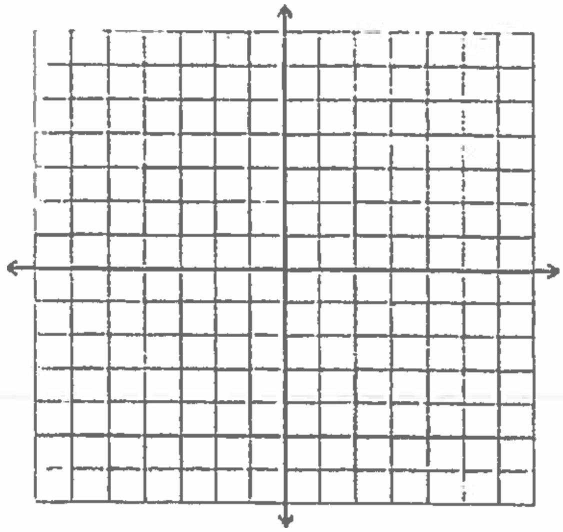
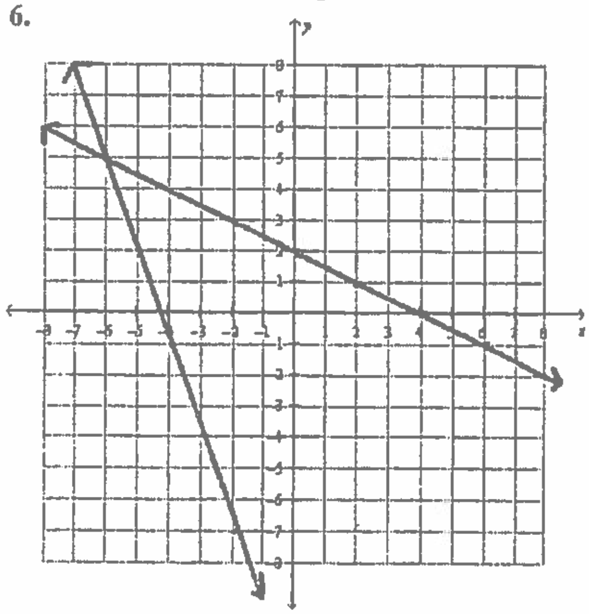
5. $\begin{cases} 2+y=2x \\ y-2x=5 \end{cases}$

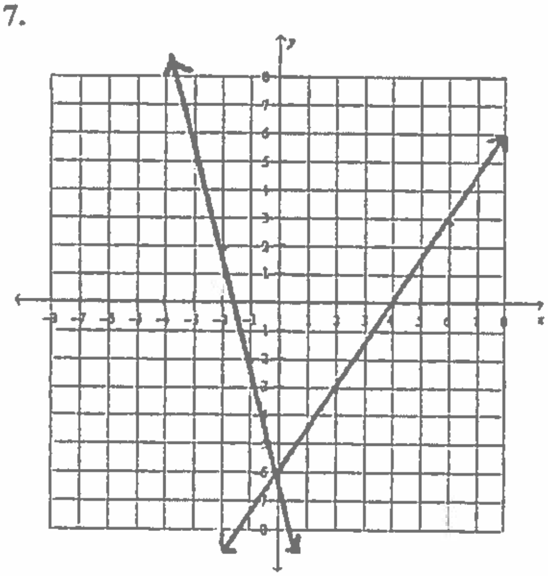


Find the solution to the given systems.

6.



7.



Spiral – Show all work:

Solve the following equations:

8. $7k-8+2(k+12)=52$

9. $6(f+5)=2(f-3)$

Name: ______________________________ Date: ______________

SOLVING SYSTEMS ALGEBRAICALLY BY SUBSTITUTION

There are a variety of ways that we solve a system of equations. In the last lesson we saw how to solve them graphically. In this lesson we will review and understand the basis for solving them by a method known as **substitution**. You have seen this technique in Common Core 8th Grade mathematics, but here we will explore it more deeply.

Exercise #1: Consider the system given below and its solution $x=4$ and $y=1$.

(a) Show that $(4,1)$ is a solution to the system.

$$2x+y=9$$

$$y=x-3$$

(b) Substitute $x-3$ in for y in the first equation and show that the point $(4,1)$ is still a solution to this new equation.

(c) Solve the system by finishing the substitution from (b).

Substitution is a very important technique and we want to be very good at it. It boils down to one of the most important properties of equality:

EQUALS MAY ALWAYS SUBSTITUTE FOR EQUALS

Exercise #2: Solve the following systems of equations by substitution.

(a) $y=2x+5$

$y=-3x-10$

(b) $4x-2y=16$

$y=-5x+13$

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Name: ______________________________ Date: ________________

SOLVING SYSTEMS BY SUBSTITUTION

FLUENCY

1. Solve each of the following system of equations by substitution.

(a) $y = x + 8$

$y = 4x - 1$

(b) $y = -3x + 5$

$2x + y = 6$

(c) $4x + 3y = 37$

$y - x = 4$

(d) $x - 5y = -49$

$y = -2x + 1$

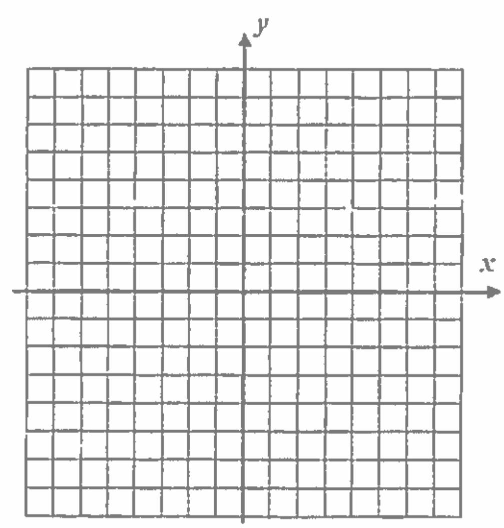
2. Given tl system shown below do the following:

$$y = \frac{1}{2}x - 2$$

$$y = -3x + 5$$

(a) Solve this system graphically using the grid shown.

(b) Solve this system by substitution. Show your work.



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Solving Systems of Equations by Substitution

Solve each system by substitution.

1) $y = -3x - 16$
$-3x - y = 16$

2) $-5x + 5y = 10$
$y = 8x - 19$

3) $6x + 3y = -9$
$y = -3$

4) $6x + 8y = 10$
$y = 5$

5) $3x - 2y = 18$
$x - 3y = 20$

6) $-2x - 3y = -1$
$x - 5y = -19$

7) $3x + y = 9$
$-6x - y = -24$

8) $x + 3y = 2$
$-4x - y = 14$

9) $x + y = 10$
$5x - 8y = 11$

10) $-x + 2y = -12$
$y = -3$

Name: ______________________________ Date: ______________

The Method of Elimination

In previous courses you have seen how to solve systems **graphically** and how to solve them by **substitution**. Today's lesson will build on the previous one and formally introduce the technique of solving a system by **elimination**. Remember from the last lesson that:

Solutions to Systems Remain Solutions If

1. Properties of equality are used to rewrite either of the equations.
2. The equations are added or subtracted or any rewrite is added or subtracted.

***Exercise* #1:** Consider the system shown below. Solve the system two ways, by eliminating x in (a) and eliminating y in (b).

(a) Eliminate x to solve

$4x+5y=12$

$-2x+y=8$

(b) Eliminate y to solve

$4x+5y=12$

$-2x+y=8$

(c) Show that the point that you found in (a) and (b) is a solution to this system of equations.

***Exercise* #2:** Solve the following system of equations by elimination and check that your answer is a solution to this system.

$5x-2y=10$

$2x+7y=43$

Name: ______________________ Date: ______________

THE METHOD OF ELIMINATION

FLUENCY

1. Solve each of the following systems by the Method of Elimination. These two should be relatively easy. Make sure to understand why.

   (a) $x - y = 7$
   $x + y = 5$

   (b) $2x + 5y = 3$
   $-2x - y = 5$

2. Solve each of the following systems by the Method of Elimination. These will be slightly harder than #1 because you will have to alter one of the equations by multiplication.

   (a) $x - y = 15$
   $4x + 2y = 30$

   (b) $2x + 3y = 17$
   $5x + 6y = 32$

3. Solve each of the following systems by the Method of Elimination. In each case you will likely want to alter both equations by multiplication.

   (a) $2x + 3y = 16$
   $5x - 2y = 21$

   (b) $6x - 7y = 25$
   $15x + 3y = 42$

4. Which of the following represents the intersection of the lines whose equations are given below?

(1) $(-1,16)$ (3) $(3,8)$

(2) $(4,9)$ (4) $(0,7)$

$$y+2x=14$$
$$y-x=5$$

APPLICATIONS

5. Use the Method of Elimination to find the equation of the line, in $y=mx+b$ form, that passes through each set of points. Set up a system first, like we did Exercises #3 and 4 from the lesson. Then, solve the system for the slope, m, and the y-intercept, b.

(a) $(3,10)$ and $(5,18)$ (b) $(-2,5)$ and $(6,-7)$

6. Lilly and Rosie are sisters. The sum of their ages is 19 and the positive difference of their ages is 9. Set up a system of equations involving Lilly's age, L, and Rosie's age, R, assuming that Lilly is the older child. Solve the system to find their ages.

7. Shana bought sodas and popcorn for the movies. Sodas cost \$3 each and popcorn cost \$4 per bag. Shana bought 7 things from the concession, all either sodas or bags of popcorn. Shana spent a total of \$26. Write a system of equations involving the number of sodas, s, and the bags of popcorn, b. Solve the system to see how many of each Shana bought.

Solving Systems of Equations by Elimination

1) $-4x - 2y = -12$
$4x + 8y = -24$

2) $4x + 8y = 20$
$-4x + 2y = -30$

3) $x - y = 11$
$2x + y = 19$

4) $-6x + 5y = 1$
$6x + 4y = -10$

5) $-2x - 9y = -25$
$-4x - 9y = -23$

6) $8x + y = -16$
$-3x + y = -5$

7) $-6x + 6y = 6$
$-6x + 3y = -12$

8) $7x + 2y = 24$
$8x + 2y = 30$

9) $5x + y = 9$
$10x - 7y = -18$

10) $-4x + 9y = 9$
$x - 3y = -6$

11) $-3x + 7y = -16$
$-9x + 5y = 16$

12) $-7x + y = -19$
$-2x + 3y = -19$

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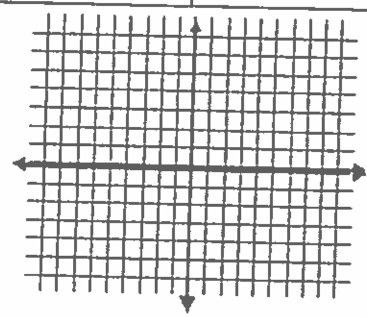
Methods for Solving Systems of Equations

$$2x + 3y = 5$$
$$4x - y = 17$$

GRAPHING



Intersecting Lines:

Parallel Lines:

Coinciding Lines:

SUBSTITUTION

ELIMINATION

What if both of the variables cancel out? Look at the resulting arithmetic equation.
* False statement indicates the lines are parallel so ______________.
* True statement indicates the lines coincide so ______________.

Name: ______________________ Date: ____________

MODELING WITH SYSTEMS OF EQUATIONS

Many real world scenarios can be modeled using systems of equations. In fact, when we have two quantities that are related and two ways in which those quantities are related, then we can often set up and solve a system.

Exercise #1: Jonathan has nine bills in his wallet that are all either five-dollar bills or ten-dollar bills.

(a) Fill out the following table to see the dependence of the two variables and how they then determine how much money Jonathan has.

Number of fives, f	Number of tens, t	Amount of Money, \$
0		
1		
2		
3		

(b) If f represents the number of \$5 bills and t represents the number of \$10 bills, then what does the following expression calculate? Explain.

$$5f+10t$$

(c) If Jonathan has a total of \$55, set up a system of equations involving f and t that could be used to determine how many of each bill he has. Solve the system. Remember that he has 9 total bills.

(d) Let's say that we were told that Jonathan had seven bills that were all 5's and 20's and we were also told that he had a total of \$120. Set up and solve a system to help evaluate whether we could have been told true information.

There are many different problems that can be modeled with linear systems. Let's try another one where we use information given to determine **unit prices**.

Exercise #2: Samantha went to a concession stand and bought three pretzels and four sodas and paid a total of \$11.25 for them. Raza went to the same stand and bought five pretzels and two sodas and paid a total of \$8.25.

(a) Could pretzels have cost \$1.75 each and sodas \$1.50 each? How can you evaluate based on the information given?

(b) Letting x equal the **unit cost** of a pretzel and letting y equal the **unit cost** of a soda, write a system of equations that models the information given.

(c) Solve the system of equations using the elimination method.

(d) If Leah went to the same concession stand and bought two sodas, how many pretzels would she need to buy so that she spent the same amount on both?

We can model information given in a geometric form as well. We should feel relatively comfortable working with rectangles and their perimeters. The next question concerns the relationship between the length and width of a rectangle.

Exercise #3: A rectangle has a perimeter of 204 feet. It's length is six feet longer than twice its width. If L stands for the length of the rectangle and W stands for its width, write a system of equations that models the information given in this problem and solve it to find the length and width of this rectangle.

Name: ______________________ Date: ______________

MODELING WITH SYSTEMS OF EQUATIONS

APPLICATIONS

1. A local theater is showing an animated movie. They charge \$5 per ticket for a child and \$12 per ticket for an adult. They sell a total of 342 tickets and make a total of \$2550. We want to try to find out how many of each type of ticket they sold. Let c represent the number of children's tickets sold and a represent the number of adult tickets sold.

   (a) Write an equation that represents the fact that 342 total tickets were sold.

   (b) Write an equation representing the fact that they made a total of \$2550.

   (c) Solve the system you created in (a) and (b) by the Method of Elimination.

2. A catering company is setting up tables for a big event that will host 764 people. When they set up the tables they need 2 forks for each child and 5 forks for each adult. The company ordered a total of 2992 forks. Set up a system of equations involving the number of adults, a, and the number of children, c, and solve to find out how many of each attended the event.

3. Ilida went to Minewaska Sate Park one day this summer. All of the people at the park were either hiking or bike riding. There were 178 more hikers than bike riders. If there were a total of 676 people at the park, how many were hiking and how many were riding their bikes?

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4. Juanita and Keenan own a camping supply store and just put in an order for flashlights and sleeping bags. The number of flashlights ordered was five times the number of sleeping bags. The flashlights cost $12 each and the sleeping bags cost $45 each. If the total cost for the flashlights and sleeping bags was $1785, how many flashlights and how many sleeping bags did Juanita and Keenan order?

5. For a concert, there were 206 more tickets sold at the door than were sold in advance. The tickets sold at the door cost $10 and the tickets sold in advance cost $6. The total amount of sales for both types of tickets was $6828. How many of *each* type of ticket was sold?

6. Eldora and Finn went to an office supply store together. Eldora bought 15 boxes of paper clips and 7 packages of index cards for a total cost of $55.40. Finn bought 12 boxes of paper clips and 10 packages of index cards for a total cost of $61.70. Find the cost of one box of paper clips and the cost of one package of index cards.

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GRAPHS OF LINEAR INEQUALITIES

So, we have graphed linear functions and in the last lesson learned that the points that lie on a graph are simply the (x, y) pairs that make the equation true. Graphing an inequality in the xy-plane is exactly the same

GRAPHING INEQUALITIES

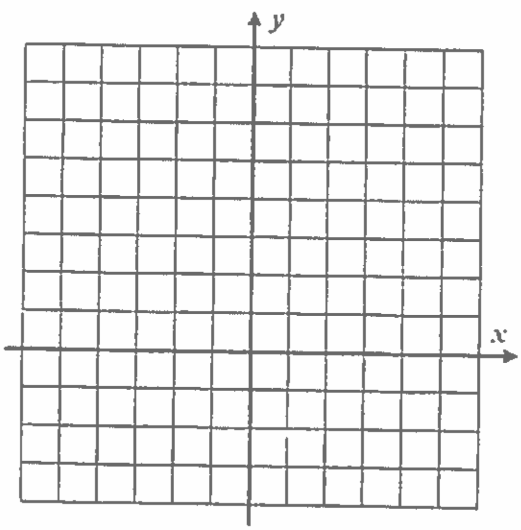
To graph an inequality simply means to plot (or shade) all (x, y) pairs that make the inequality true

***Exercise* #1:** Consider the inequality $y > x + 3$.

(a) Determine whether each of the following points lies in the solution set (and thus on the graph of) the inequality given.

$(2, 7)$ $\qquad\qquad$ $(0, 1)$ $\qquad\qquad$ $(1, 4)$

(b) Graph the line $y = x + 3$ on the grid below in **dashed form**. Why are points that lie on this line **not** part of the **solution set** of the inequality?



(c) Plot the three points from part (a) and use them to help you shade the proper region of the plane that represents the solution set of the inequality.

(d) Choose a fourth point that lies in the region you shaded and show that it is in the solution set of the inequality.

(e) The point $(10, 12)$ cannot be drawn on the graph grid above, so it is difficult to tell if it falls in the shaded region. Is $(10, 12)$ part of the solution set of this inequality? Show how you arrive at your answer.

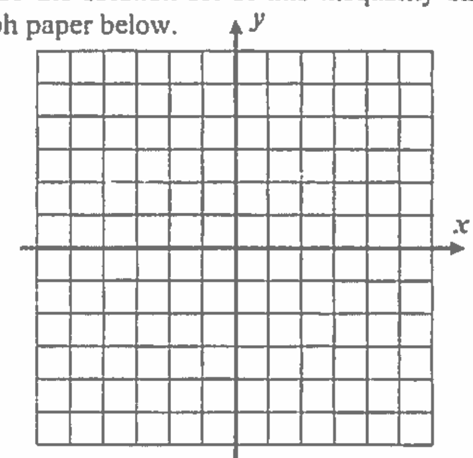
There are some challenges to graphing linear inequalities, especially if the output, y, has not been solved for. Let's look at the **worst case scenario.**

Exercise **#2:** Consider the inequality $3x-2y\geq 2$

(a) Rearrange the left-hand side of this inequality using the commutative property of addition.

(b) Solve this inequality for y by applying the **properties of inequality** that we used in Unit #2.

(c) Shade the solution set of this inequality on the graph paper below.



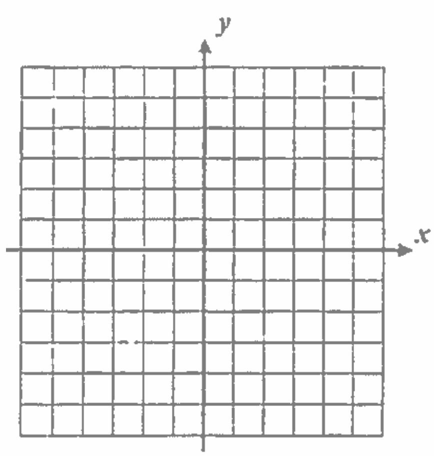
(d) Pick a point in the shaded region and show that it is a solution to the **original** inequality.

The final type of inequality that we should be able to graph quickly and effectively is one that involves either a **horizontal line** or a **vertical line.**

Exercise **#3:** Shade the solution set for each of the following inequalities in the xy-planes provided. First, state in your own words the (x, y) pairs that the inequality is describing.

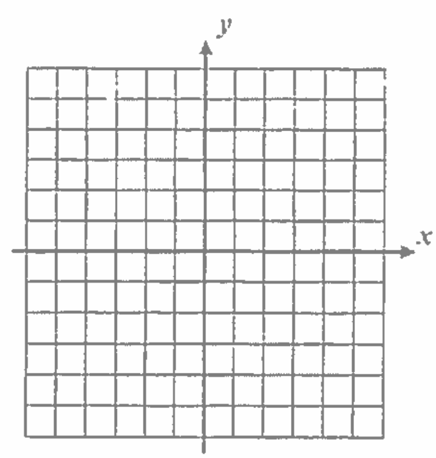
(a) $x<4$

Your own words:



(b) $y\geq -2$

Your own words:



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GRAPHS OF LINEAR INEQUALITIES

FLUENCY

1. Determine which of following points lie in the solution set of the inequality $y \geq 2x-4$ and which do not. Justify each choice.

   (a) $(5, 4)$ (b) $(0, -1)$

   (c) $(10, 16)$ (d) $(2, -1)$

2. Which of the following points lies in the solution set of the inequality $y \geq 3x+10$?

   (1) $(1, 10)$ (3) $(4, 20)$

   (2) $(-1, 3)$ (4) $(2, 16)$

3. Which of the following points does *not* lie in the solution set to the inequality $y \geq -\frac{1}{3}x+5$?

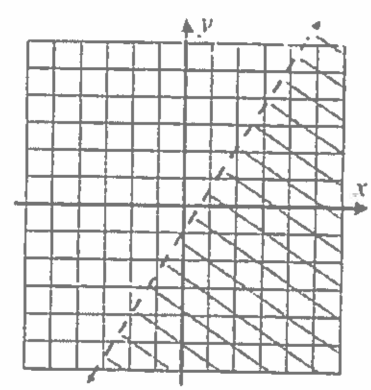
   (1) $(6, 3)$ (3) $(-3, 8)$

   (2) $(-6, 5)$ (4) $(12, 3)$

4. Which of the following linear inequalities is shown graphed below?

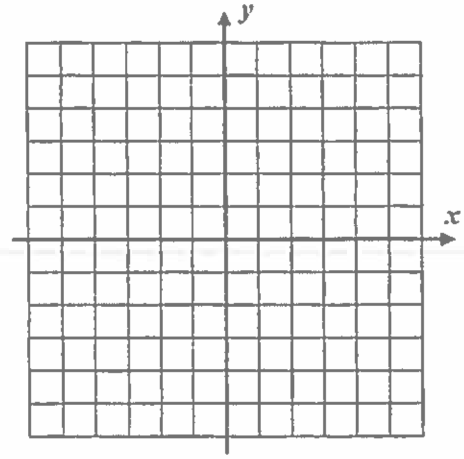
   (1) $y < \frac{3}{2}x-1$ (3) $y > \frac{2}{3}x-1$

   (2) $y \leq \frac{2}{3}x-1$ (4) $y \geq \frac{3}{2}x-1$



5. Graph the solution set to the inequality shown below. State one point that lies in the solution set and one point that does not.

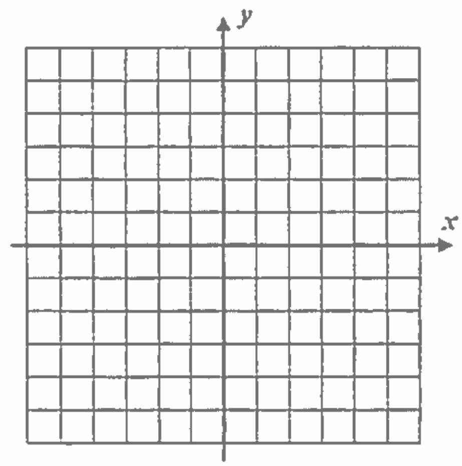
$y < -2x + 4$



One Point In Solution: One Point Not In Solution:

6. Rearrange the inequality below so that it is easier to graph and then sketch its solution set on the grid given. Be careful when dividing by a negative and remember to switch the inequality sign.

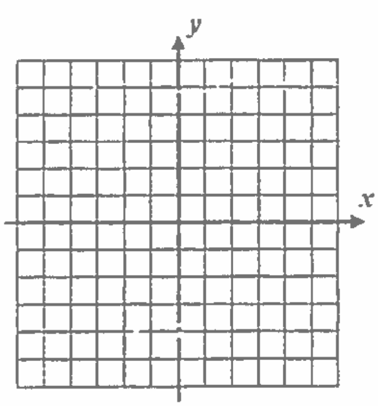
$x - 2y \leq 6$



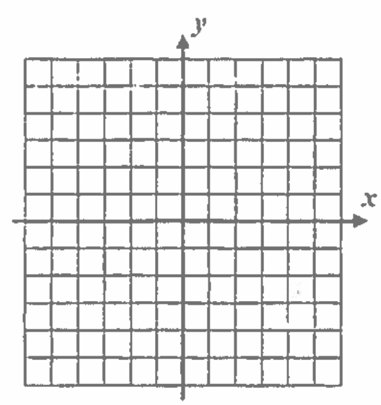
One Point In Solution: One Point Not In Solution:

7. Graph the solution set to each of the following inequalities.

(a) $y \leq 4$



(b) $x > 1$



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SYSTEMS OF INEQUALITIES

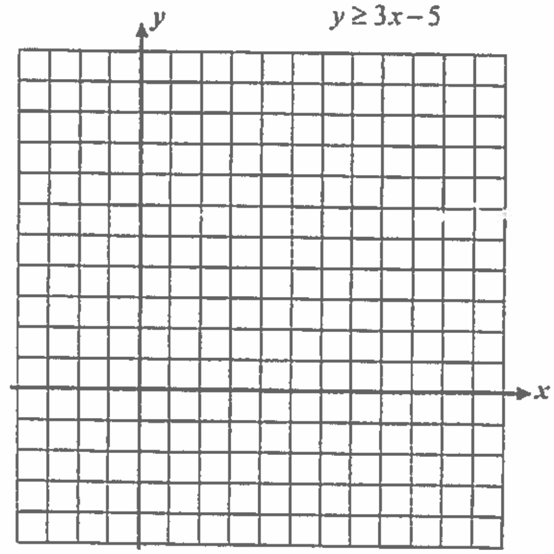
We can have **systems of inequalities** as well as **systems of equations (equalities)**. The definition of solving a system still holds: we have to find all points that make all inequalities true.

Exercise #1: Consider the system of inequalities shown below. Determine if each of the following points is a solution or not to the system. Show work that justifies your answers.

(a) $(3,8)$

(b) $(5,9)$

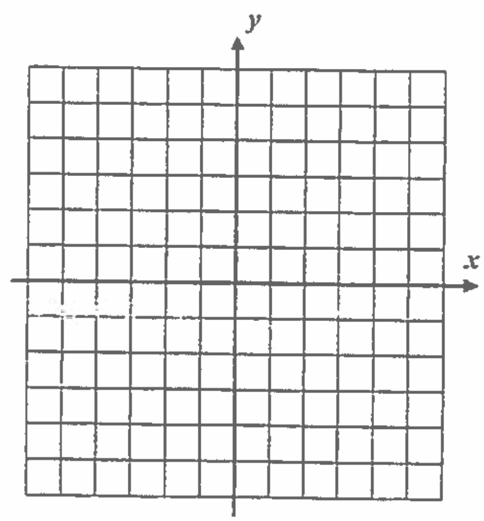
$$x+y>10$$
$$y\geq 3x-5$$



(c) Graph the solution set to this system of inequalities.

Exercise #2: On the grid shown below, graph the solution to the system of inequalities shown below. State a point that lies in the solution set and one that doesn't.

$$y<-\frac{3}{2}x+2$$
$$x\geq -2$$



Point in Solution Set:

Point Not in Solution Set:

Exercise #3: Which of the following points is a solution to the system of inequalities shown below? Show the work that leads to your answer.

(1) $(3,-6)$ (3) $(-2,10)$

(2) $(0,2)$ (4) $(4,10)$

$$y \le -4x+2$$

$$y > \frac{x}{2}+7$$

Very often, systems of inequalities will define portions of the xy-plane that can be visualized and manipulated.

Exercise #4: Consider the system of inequalities given below.

$$y \ge -2$$

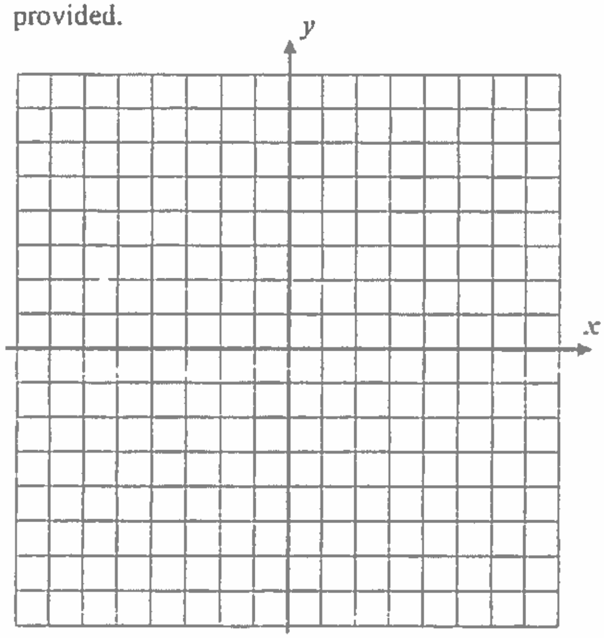
$$x < 4$$

$$y \le 2x$$

(a) Determine which, if any, of these points is a solution to the system.

$(-1,4)$ $(3,1)$

(b) Sketch the solution to the system on the grid provided.



(c) Find the area of the portion of the xy-plane that represents the solution.

(d) Why does the dashed line of one of the borders not make a difference in terms of the area you found in part (c)?

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Solving Systems of Inequalities

Fluency

1. Which of the following points is a solution to the system of inequalities shown below?

(1) $(3,5)$ (3) $(1,-2)$

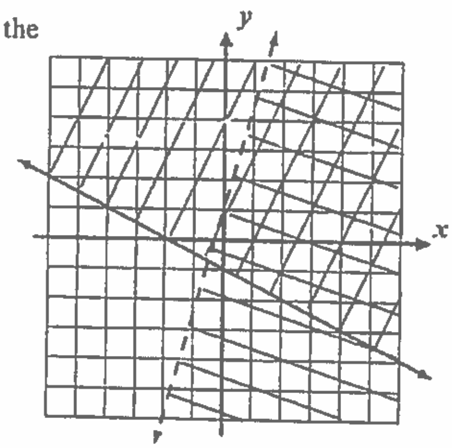
(2) $(1,3)$ (4) $(2,3)$

$$y > x + 1$$
$$y \le -2x + 7$$

2. A system of inequalities is shown graphed below. Which of the following points lies in the solution set of this system?

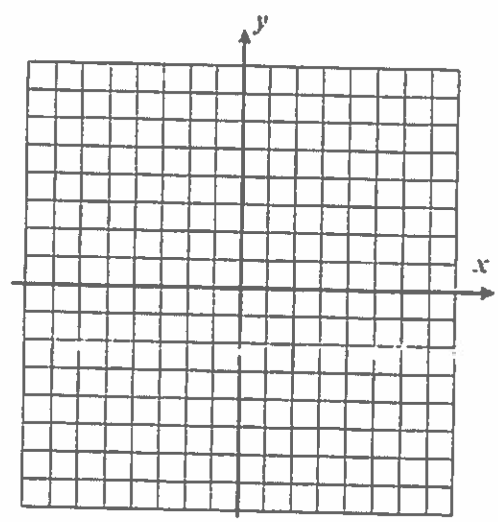
(1) $(-1,2)$ (3) $(2,-4)$

(2) $(1,5)$ (4) $(4,2)$



3. Consider the system of inequalities shown below.

$$y > \frac{2}{3}x - 2$$
$$y \le -x + 6$$



(a) Is the origin, $(0,0)$, part of the solution set of the system? Determine without first graphing.

(b) Graph the solution to the system of inequalities. Then, state one point that lies in the set and one that doesn't.

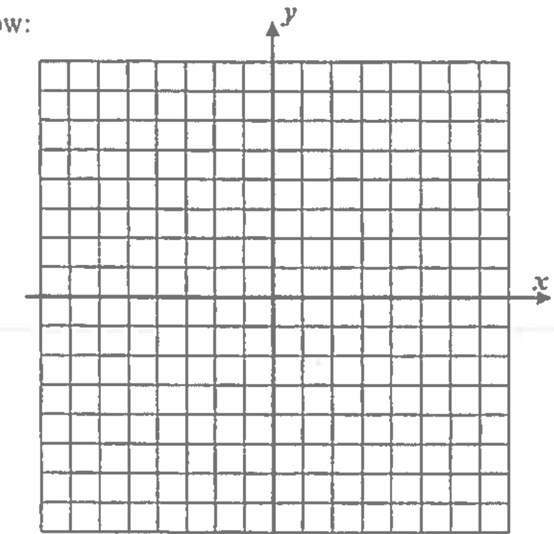
One Point That Lies in the Solution: One Point that Does Not Lie in the Solution

4. Sketch the solution to the system of inequalities shown below:

$y + 2x < 6$

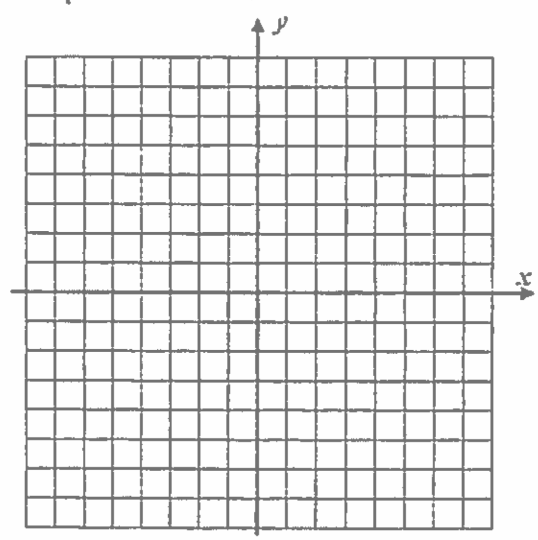
$x \leq 2$

State a point that lies in the solution set:



5. Find the area of the triangular region defined by the system of inequalities shown below.

$y \geq x$
$x \geq -3$
$y \leq 6$



REASONING

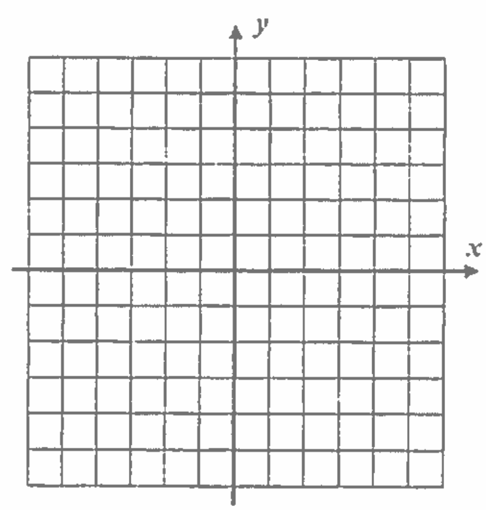
6. Consider the system of inequalities shown below:

$y \geq x + 2$

$y \leq x - 3$

(a) Graph the system solution to the system on the grid.

(b) Why can you not state a point in the solution set?



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MODELING WITH SYSTEMS OF INEQUALITIES

There are many situations that arise in business and engineering that necessitate systems of linear inequalities. The **region in the** *xy*-**plane** that **solves the systems** often represents all of the **viable solutions** to the system, so being able to visualize this region can be extremely helpful. As always, with modeling, it is important to really read the problems and understand the physical quantities involved.

Exercise #1: John mows yards for his father's landscaping business for \$10 per hour and also works at a bakery for \$15 per hour. He can work at most 52 hours per week during the summer. He needs to make at least \$600 per week to cover his living expenses.

(a) If John works 14 hours mowing and 30 hours at the bakery, does this satisfy all of the problem's **constraints**?

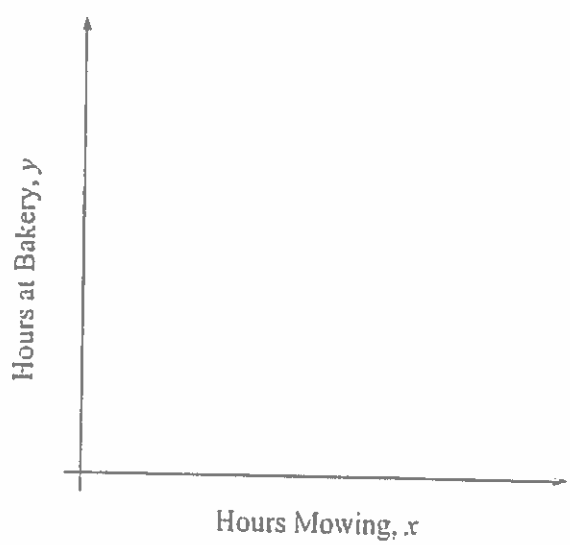
(b) If x represents the hours John spends mowing and y represents the hours he spends at the bakery, write a system of inequalities that describes this scenario.

(c) If John must work a minimum of 10 hours for his father, will he be able to make enough money to cover his living expenses? Show the work that leads to your answer.

(d) Graph the system of inequalities with the help of your calculator (if needed) on the axes below. Use the space below to think about how to graph these lines.

(e) John's father needs him to work a lot at the landscaping business. Show the point on the graph that corresponds to the greatest number of hours that he can work while still covering his expenses.

(f) Algebraically, find the greatest number of hours that John can work for his father and still cover his expenses. Explain how you found your answer or show your algebra below.



Exercise #2: For each of the following, write a system of inequalities that models the problem. You do not need to solve the system.

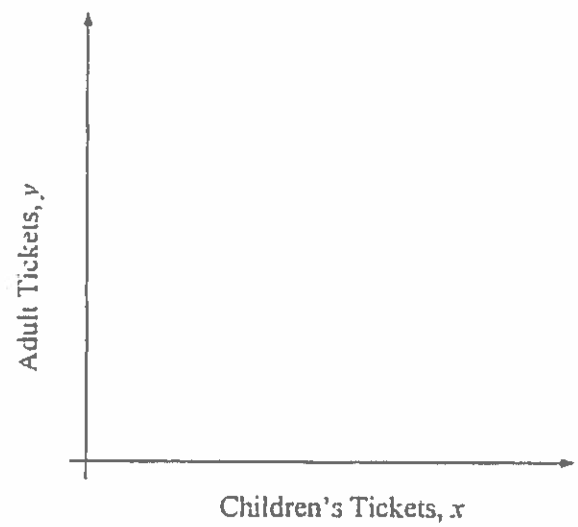
(a) Frank is putting together a bouquet of roses and daisies. He wants at least one rose and at least two more daisies than roses. Roses cost $4 each and daisies cost $2 each. Frank must spend $40 or less on this bouquet. If r represents the number of roses he buys and d represent the number of daisies, write the system.

(b) A diet food company is attempting to create a non-carb brownie composed entirely of fat and protein. The brownie must weigh at least 10 grams but have no more than 100 calories. Fat has 9 calories per gram and protein has 4 calories per gram. If x represent the weight, in grams, of protein and y represents the weight, in grams, of fat, write the system.

Exercise #3: The drama club at a local high school is trying to raise money by putting on a play. They have only 500 seats in the auditorium that they are using and are selling tickets for these seats at $5 per child's ticket and $10 per adult ticket. They must sell at least $2000 worth of tickets to cover their expenses.

(a) If x represents the number of children's tickets sold and y represents the number of adult tickets sold, write a system of inequalities that models this situation.

(b) Using technology, sketch the region in the coordinate plane that represents solutions to this system of inequalities.



(c) If the students want to sell exactly 500 tickets and make exactly $2000, how many of each ticket should they sell? Why is this answer not realistic?

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Modeling with Systems of Inequalities

Applications

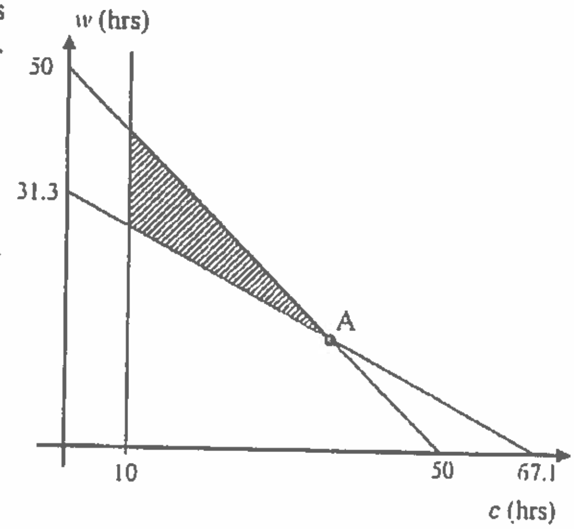
1. Jody is working two jobs, one as a carpenter and one as a website designer. He can work at most 50 hours per week and makes \$35 per hour as a carpenters and \$75 an hour as a website designer. He wants to make at least \$2350 per week but also wants to work at least 10 hours per week as a carpenter. Let c represent the hours he works as a carpenter and let w represent the hours he works as a website designer.

   (a) Write a system of inequalities that models this scenario.

   (b) What is the maximum amount of money that Jody can make in a week given the system in (a)? Explain your reasoning.

   (c) The graph of the system is shown below with its solutions shown shaded. Three lines are graphed. Label each with its equation.

   (d) Find the coordinates of point A by solving a system of equations by Elimination.



   (e) What does the value of c that you found in the solution to part (d) represent about the number of hours Jody can work as a carpenter. Explain your thinking.

2. For each of the following, create a system of inequalities that models the scenarios presented. You do not need to solve the systems.

   (a) Two pumps at a local water facility can only run individually. They will run for at least 18 hours in a day but obviously no more than 24 hours in a day. Pump 1 can move 120 gallons per hour while Pump 2 can move 200 gallons per hour. In total the two pumps must move at least 3,000 gallons of water per day. If x represents the number of hours that Pump 1 runs and y represents the number of hours that Pump 2 runs, write a system of inequalities that models all conditions.

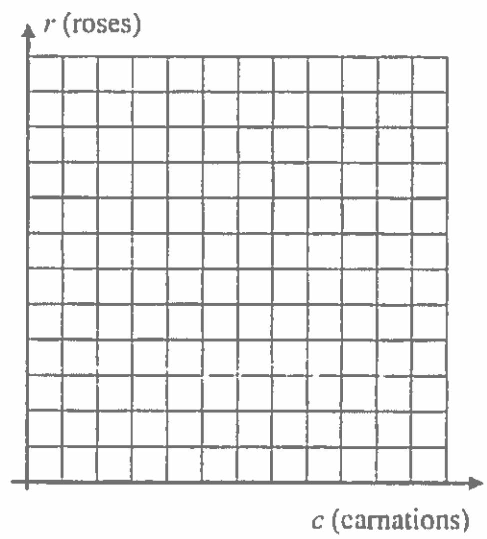
   (b) Dave is buying popcorn and sodas for his son and his three friends that he brings to the movies (four kids total). He needs to buy at least one of the two items for each of the four. Popcorn costs \$2.50 per bag and sodas cost \$4.00 each. Dave can spend at most \$20. If s represents the number of sodas he buys and p represents the number of bags of popcorn, then write a system that models this scenario.

REASONING

3. Systems of inequalities can also come in **discrete versions** where the two variables involved can only take on **integer values**. Let's look at a simple example of this.

   Jennifer is putting together a selection of flowers that has at most 12 flowers in it. She is choosing either roses or carnations. She wants to pick at least three roses and at least two carnations. Let r be the number of roses she uses and let c be the number of carnations she uses.

   (a) Write a system of inequalities that models this scenario.



   (b) If Jennifer used the minimum number of carnations, what is the maximum number of roses she could use?

   (c) What is the fewest flowers Jennifer will use and in what combination?

   (d) Graph the solution set to the system. Be careful, this should be a collection of points, not a shaded region.