Name: KEY

Date: _____

MODELING WITH SYSTEMS OF INEQUALITIES

There are many situations that arise in business and engineering that necessitate systems of linear inequalities. The **region** in the *xy*-plane that **solves the systems** often represents all of the **viable solutions** to the system, so being able to visualize this region can be extremely helpful. As always, with modeling, it is important to really read the problems and understand the physical quantities involved.

Exercise #1: John mows yards for his father's landscaping business for $10 per hour and also works at a bakery for $15 per hour. He can work at most 52 hours per week during the summer. He needs to make at least $600 per week to cover his living expenses.

(a) If John works 14 hours mowing and 30 hours at the bakery, does this satisfy all of the problem's constraints?

$14+30=44$ ✓

$14(10) + 15(30) = 140 + 450 = 590$

no, not enough money

(b) If x represents the hours John spends mowing and y represents the hours he spends at the bakery, write a system of inequalities that describes this scenario.

$$X + Y \leq 52$$

$$10x + 15y \geq 600$$

(c) If John must work a minimum of 10 hours for his father, will he be able to make enough money to cover his living expenses? Show the work that leads to your answer.

$10 \times 10 = 100$

$42x = 630$

$100 + 630 = 730$

it's possible

(d) Graph the system of inequalities with the help of your calculator (if needed) on the axes below. Use the space below to think about how to graph these lines.

$X + Y \leq 52$
$-x \quad -x$
$Y \leq -X + 52$

$10x + 15y \geq 600$
$-10x \quad -10x$
$\dfrac{15y}{15} \geq \dfrac{-10x + 600}{15}$

$Y \geq -\frac{2}{3}x + 40$

(e) John's father needs him to work a lot at the landscaping business. Show the point on the graph that corresponds to the greatest number of hours that he can work while still covering his expenses. Where the 2 lines intersect

(f) Algebraically, find the greatest number of hours that John can work for his father and still cover his expenses. Explain how you found your answer or show your algebra below.

if it's where they intersect,
Solve for intersection point

$-10(X + Y = 52)$
$10x + 15y = 600$
$-10x - 10y = -520$
─────────────
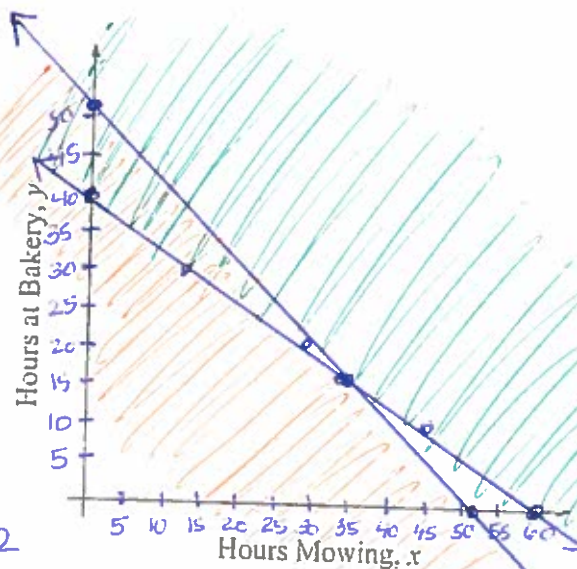$\dfrac{5y}{5} = \dfrac{80}{5}$

$Y = 16$

$X + Y = 52$
$X + 16 = 52$
$\quad -16 \quad -16$
$X = 36$

36 hours with his dad
and 16 hours at the bakery

$36(10) + 16(15)$
$360 + 240$
600 ✓

Exercise #2: For each of the following, write a system of inequalities that models the problem. You do not need to solve the system.

(a) Frank is putting together a bouquet of roses and daisies. He wants at least one rose and at least two more daisies than roses. Roses cost $4 each and daisies cost $2 each. Frank must spend $40 or less on this bouquet. If r represents the number of roses he buys and d represent the number of daisies, write the system.

$$r \geq 1$$
$$d + 2 \leq r$$
$$4r + 2d \leq 40$$

(b) A diet food company is attempting to create a non-carb brownie composed entirely of fat and protein. The brownie must weigh at least 10 grams but have no more than 100 calories. Fat has 9 calories per gram and protein has 4 calories per gram. If x represent the weight, in grams, of protein and y represents the weight, in grams, of fat, write the system.

$$x + y \geq 10$$
$$4x + 9y \leq 100$$

Exercise #3: The drama club at a local high school is trying to raise money by putting on a play. They have only 500 seats in the auditorium that they are using and are selling tickets for these seats at $5 per child's ticket and $10 per adult ticket. They must sell at least $2000 worth of tickets to cover their expenses.

(a) If x represents the number of children's tickets sold and y represents the number of adult tickets sold, write a system of inequalities that models this situation.

$$x + y \leq 500$$
$$5x + 10y \geq 2000$$

$$y \leq -x + 500$$
$$y \geq -\tfrac{1}{2}x + 200$$

(b) Using technology, sketch the region in the coordinate plane that represents solutions to this system of inequalities.
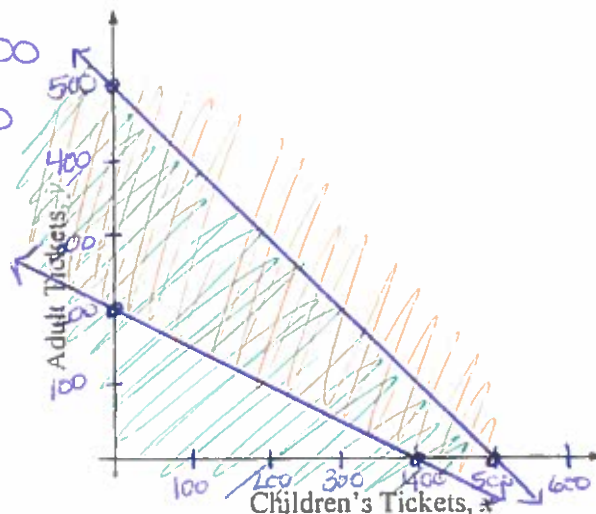
(c) If the students want to sell exactly 500 tickets and make exactly $2000, how many of each ticket should they sell? Why is this answer not realistic?

not possible- the lines don't cross in the first quadrant, and you can't sell a negative number of tickets

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MODELING WITH SYSTEMS OF INEQUALITIES

APPLICATIONS

1. Jody is working two jobs, one as a carpenter and one as a website designer. He can work at most 50 hours per week and makes $35 per hour as a carpenters and $75 an hour as a website designer. He wants to make at least $2350 per week but also wants to work at least 10 hours per week as a carpenter. Let c represent the hours he works as a carpenter and let w represent the hours he works as a website designer.

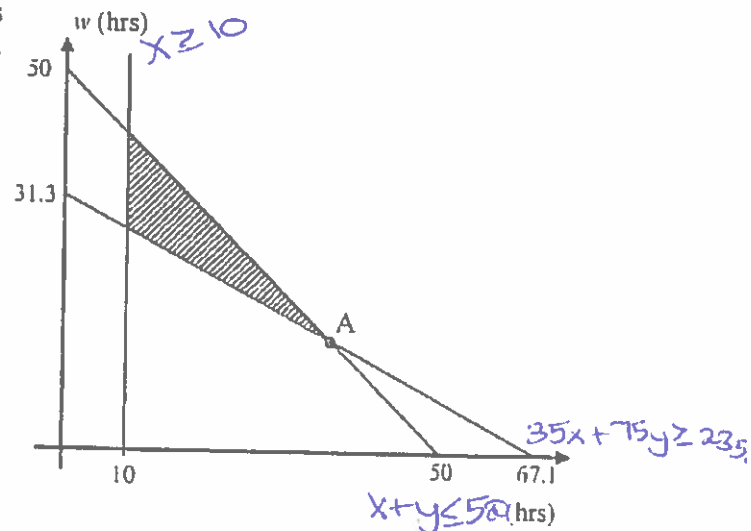(a) Write a system of inequalities that models this scenario.

x = hours as Carpenter
y = hours as web designer

$-35(x + y \leq 50)$ $x \geq 10$

$35x + 75y \geq 2350$

(b) What is the maximum amount of money that Jody can make in a week given the system in (a)? Explain your reasoning.

(c) The graph of the system is shown below with its solutions shown shaded. Three lines are graphed. Label each with its equation.

(d) Find the coordinates of point A by solving a system of equations by Elimination.

$-35x - 35y \leq -1750$
$35x + 75y \geq 2350$
_____
$\dfrac{40y}{40} = \dfrac{600}{40}$

$y = 15$

$x + 15 = 50$
$\quad -15 \quad -15$
$x = 35$

(e) What does the value of c that you found in the solution to part (d) represent about the number of hours Jody can work as a carpenter. Explain your thinking.

Jody can work 35 hours as a carpenter and 15 hours as a website designer.