2

Sequences can be defined by a classic function formula, like what we saw in Exercise #2, and they also can be defined **recursively**. A **recursive formula** is one where each term in the sequence **depends on a term or terms that came before it.**

Exercise #3: Consider a sequence of numbers given by the following definition:

$$b_1 = 7 \text{ and } b_i = b_{i-1} + 4$$

(a) Give a common sense interpretation for this **recursive** function rule.

the list starts with 7 and each term is 4 more than the previous term

(b) Write out the rule for the first 4 terms and evaluate each one of them (except b_1 which is given).

$b_1 = 7$

$b_2 = b_1 + 4 = 7+4 = 11$

$b_3 = b_2 + 4 = 11+4 = 15$

$b_4 = b_3 + 4 = 15+4 = 19$

One of the most famous of all **recursively defined** sequences is known as the **Fibonacci Sequence**. Let's play around with it in the next exercise.

Exercise #4: The Fibonacci Sequence is defined recursively as follows:

$$a(1) = 1,\ a(2) = 1 \text{ and } a(n) = a(n-1) + a(n-2)$$

(a) How do you interpret this recursive rule? Write it down in your own words.

Starts with two 1's and then each term is found by adding the previous two terms

(b) Write down the rule for $a(3), a(4)$, and $a(5)$ and determine their values.

$a(1) = 1$ and $a(2) = 1$

$a(3) = a(1) + a(2) = 1+1 = 2$

$a(4) = a(2) + a(3) = 1+2 = 3$

$a(5) = a(3) + a(4) = 2+3 = 5$

Sequences often show up in the real world, where they are sometimes defined in terms of a recursive process.

Exercise #5: Kirk is trying to train for the marathon. His first month, he runs 5 miles per workout. He adds an additional 3 miles to his workout for each month that he trains.

(a) Fill out the table below for the amount of miles he runs as a function of how many months he has been running.

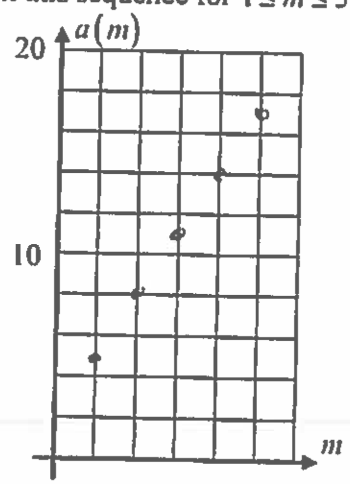
m	1	2	3	4	5
$a(m)$	5	8	11	14	17

(b) Give a **recursive definition** for the sequence $a(m)$. Don't forget to give an initial value.

$a(1) = 5$

$a(m) = a(m-1) + 3$

(c) Graph this sequence for $1 \le m \le 5$.



Name: KEY   Date: ____________

INTRODUCTION TO SEQUENCES

A sequence is a very special type of function. When students first encounter sequences, they often think of them as just a list of numbers in some particular order (and then they have to find the pattern). We will start with the technical definition of a sequence in terms of a function.

> **SEQUENCE DEFINITION**
>
> A sequence is a function whose set of inputs, the **domain**, is a subset of the natural numbers, i.e. $\{1, 2, 3, 4, \ldots\}$. A sequence is often shown as an ordered list of numbers, called the **terms or elements of the** sequence. Sequence function notation can be tricky.

***Exercise* #1:** Consider the sequence below. If we represent this sequence with the letter a then do the following.

$$4, 8, 16, 32, 64, 128, 256$$

(a) Find $a(3)$

$a(3)=16$

(b) Find $a(1)+a(7)$

$4+256=260$

(c) Find a_2.

8

(d) Find $(a_1)^2$

$4^2=16$

(e) Find a_5-a_4

$64-32=32$

(f) Solve for n: $a(n)=128$.

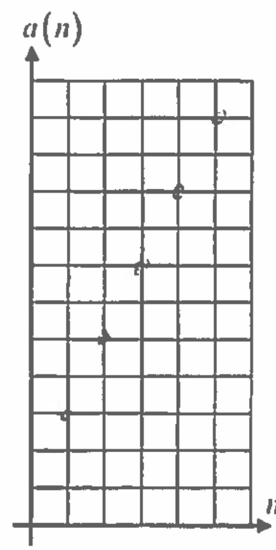
$n=7$

Sequences are functions. The key here is that the input is simply the **number's place in line** so to speak and the output is the actual **number in the list.**

***Exercise* #2:** Consider the sequence defined by the formula $a(n)=2n+1$.

(a) Write out the first 5 elements of this sequence.

$a(1)=2(1)+1=3$
$a(2)=2(2)+1=5$
$a(3)=2(3)+1=7$
$a(4)=2(4)+1=9$
$a(5)=2(5)+1=11$



(b) Graph the sequence on the grid shown below for $1 \le n \le 5$.

(c) Why shouldn't we connect the points plotted with a continuous straight line?

Sequences only include whole numbers, starting with 1

(d) What is the 21st term of this sequence? Show how you arrived at your answer.

$a(21)=2(21)+1=43$