Name: KEY Date: ____________

EXPONENTIAL FUNCTIONS

So far we have concentrated on **linear functions** which are characterized by having a **constant rate of change.** In the last lesson, we looked at **exponential growth and decay**. In this lesson we will more formally introduce the concept of an **exponential function.**

Exercise **#1:** Consider the exponential function $f(x)=8(2)^x$. Answer the following.

(a) Evaluate each of the following and indicate what point must lie on the graph of $f(x)$ based on each:

(i) $f(2)= 8\cdot 2^2 = 32$

(ii) $f(0)= 8\cdot 2^0 = 8$

(iii) $f(-1)= 8\cdot 2^{-1} = 4$

(b) Calculate the average rate of change of f over the interval $-1\le x\le 0$.

$$\frac{\Delta y}{\Delta x}=\frac{4-8}{-1-0}=\frac{-4}{-1}=4$$

(c) Calculate the average rate of change over the interval $0\le x\le 2$.

$$\frac{\Delta y}{\Delta x}=\frac{8-32}{0-2}=\frac{-24}{-2}=12$$

(d) What does comparing answers from (b) and (c) tell you about this function? Explain.

not a linear function because the average rate of change does not stay the same

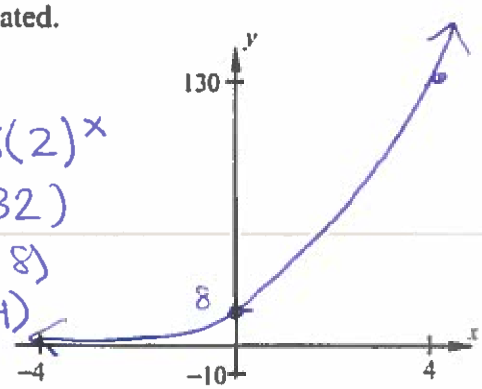
(e) Using your calculator, draw a sketch of this function on the axes below using the window indicated.

$y=8(2)^x$

(2,32)

(0,8)

(-1,4)



Exponential functions are all about **multiplication.** The basic form of an exponential function is given below.

EXPONENTIAL FUNCTIONS

A general exponential function has the form: $y=a(b)^x$, where a is the y-**intercept** and b is the base or **multiplying factor.** Sometimes b is known as the **growth factor.**

initial value

multiplier

20

Let's work some more with exponential functions to develop a better sense for them.

***Exercise* #2:** Consider the function $g(x)=54\left(\frac{1}{3}\right)^x$.

(a) Evaluate $g(0)$. What point does this indicate on the graph of g?

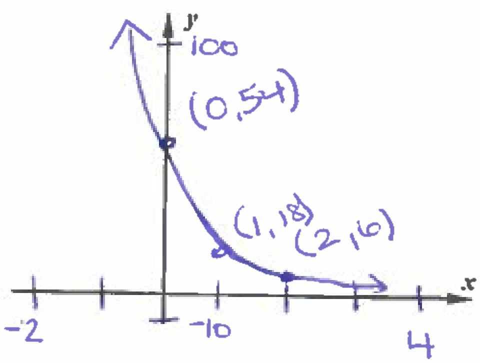
$g(0)=54\left(\frac{1}{3}\right)^0=54(1)=54$

(b) Without the use of your calculator, determine the values of $g(1)$ and $g(2)$.

$g(1)=54\left(\frac{1}{3}\right)^1=54\left(\frac{1}{3}\right)=18$ $(1,18)$

$g(2)=54\left(\frac{1}{3}\right)^2=54\left(\frac{1}{9}\right)=6$ $(2,6)$

(c) Using your graphing calculator, sketch a graph of this function using the WINDOW $-2\le x\le 4$ and $-10\le y\le 100$. Mark the y-intercept.



(d) Why is this exponential function always decreasing while the one in Exercise #1 is always increasing?

INCREASING VS. DECREASING EXPONENTIALS
$y=a(b)^x$ will increase if $b>1$
$y=a(b)^x$ will decrease if $0<b<1$

***Exercise* #3:** For each of the following exponential functions, give its y-intercept and tell whether it is increasing or decreasing.

(a) $y=8\left(\frac{2}{3}\right)^x$ $(0,8)$ decreasing ↳ $0<\frac{2}{3}<1$

(b) $f(x)=125(1.5)^x$ $(0,125)$ increasing ↳ $1.5>1$

(c) $P(t)=56\left(\frac{3}{2}\right)^t$ $(0,56)$ increasing ↳ $\frac{3}{2}>1$

The equations of exponential functions are relatively easy to determine, if you understand this lesson so far. See what you can do in the next exercise.

***Exercise* #4:** Find the equation of the exponential function, in $y=a(b)^x$ form, for the function given in the table below. Show or explain your thinking.

x	0	1	2	3	4
y	10	30	90	270	810

×3

$a=10$

$b=3$

$y=10(3)^x$